

# The Lunar Laser Ranging Experiment

Accurate ranges have given a large improvement in the lunar orbit and new selenophysical information.

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The lunar laser ranging experiment had its origins in the late 1950's in the gravitational research program at Princeton University. R. H. Dicke and his co-workers were considering ways to look for possible slow changes in the gravitational constant  $G$  by precision tracking of a very dense artificial satellite in a high-altitude orbit. The use of optical retroreflectors on the satellite and pulsed searchlight illumination from the ground to measure angular motion with respect to the stars was one of the methods considered in detail (1). When pulsed ruby lasers were developed, it became clear that laser range measurements to retroreflectors on artificial satellites and on the moon would provide much more accurate tracking information. In a written discussion of lunar laser ranging prepared at Princeton in 1962 (2), the possibility of a semisoft landing of a retroreflector by one of the Ranger missions was suggested and a picture of a cube corner reflector mounted in a self-righting silicone rubber package was included.

In 1962, Smullin and Fiocco (3) at Massachusetts Institute of Technology succeeded in observing laser light pulses reflected from the lunar surface using a laser with millisecond pulse length. Additional measurements of this kind were reported by Grasyuk *et al.* (4) from

the Crimean Astrophysical Observatory, and later Kokurin *et al.* reported successful results (5) using a Q-switched ruby laser. Plans for the use of corner reflectors on an artificial satellite were described by Plotkin (6) in 1963. Successful satellite range measurements were obtained soon after the Explorer XXII satellite was launched in 1964.

The scientific objectives achievable through high-accuracy range measurements to lunar retroreflectors, as perceived in 1964, included the following: (i) a much improved lunar orbit; (ii) determination of the location of the retroreflectors with respect to the lunar center of mass; (iii) study of the lunar physical librations (angular motions about the center of mass due to gravitational torques on the moon); (iv) determinations of the locations of ground stations on the earth from which range observations were made; and (v) an accurate check on gravitational theory, through a search for deviations from the calculated range after all known parameters in the problem had been adjusted. The advantages of optical retroreflectors for lunar distance measurements (7) were enumerated in 1965, and a proposal for the experiment (8) was submitted to the National Aeronautics and Space Administration (NASA) the same year.

A specific study of design questions related to the operation of retroreflectors on the lunar surface indicated that a reflector panel containing a number of solid fused silica corner reflectors roughly 4 centimeters in diameter would be capable of maintaining essentially diffraction limited performance under direct solar illumination and despite the severe temperature changes that take place on the lunar surface (9). Strong emphasis was placed on the importance of the retroreflectors being designed to operate even during the lunar day, in order to avoid the loss of data during the illuminated half of each month. A successful test of the performance of solid fused silica corner reflectors under simulated lunar surface conditions with direct solar illumination was conducted in 1966 (10).

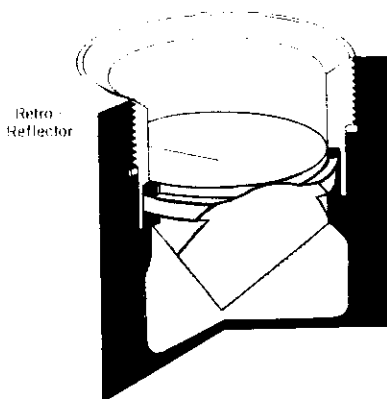
The future prospects for the lunar ranging experiment were still in doubt in the fall of 1968. The possibility of retroreflectors being carried on an unmanned NASA mission disappeared with the close of the Surveyor program. The Apollo Lunar Surface Experiments Package (ALSEP) for each of the early Apollo lunar landing missions was under construction, and space for a reflector panel could not be made available without scheduled experiments being omitted. It was thus with great excitement that we learned on 6 September that the astronaut work load for the first manned landing by Apollo 11 in July 1969 might be too heavy to permit deployment of the planned ALSEP. A proposal to develop the lunar ranging retroreflectors (LR<sup>3</sup>) as a contingency experiment for Apollo 11 was quickly prepared. Three factors led to the final decision to carry the LR<sup>3</sup> on the Apollo 11 flight: the importance of

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Fig. 1. Cutaway view of a cube corner retroreflector mounted in the Apollo 11 package. Two cube corner reflectors are made by cutting a nearly perfect cube of fused silica in half across a body diagonal, and then polishing the resulting new faces flat. Light entering the front face is reflected from all three mutually perpendicular rear faces of the corner, and then goes back in the same direction it came from.



the scientific objectives which could be achieved; the reliability inherent in the completely passive nature of the retroreflector array; and the very short time required for deployment by the astronauts on the lunar surface.

### Initial Apollo 11 Observations

The Apollo 11 reflector package contains 100 solid fused silica corner reflectors (see Fig. 1) mounted in a 46 cm square aluminum panel. The reflectors are 3.8 cm in diameter and are recessed by 1.9 cm into circular holes in the panel for thermal control purposes. The reflector panel can be tipped so that it points roughly toward the earth and maximizes the effective cross-sectional area. The optical librations of the moon (that is, the apparent rotations of the moon caused by our observing it from somewhat different directions at different times) cause light transmitted from the earth to hit

the reflectors at angles of up to  $11^\circ$  to the normal. The illuminated spot on the moon is typically 4 to 6 kilometers in diameter but, because each corner reflector sends the light hitting it back in almost the same direction from which it comes, the return signal at the earth from the reflector panel is 10 to 100 times larger than the reflected intensity from the lunar surface.

We were fortunate in obtaining assistance both in setting up and running the experiment from the Lick Observatory of the University of California and the McDonald Observatory of the University of Texas. The Lick 3.0-meter (120-inch) telescope located on Mt. Hamilton in California was made available for the initial acquisition period soon after the Apollo 11 landing, and the 2.7-m (107-inch) telescope at

the McDonald Observatory was made available for a regular program of observation extending over a number of years. Since work at the Lick and McDonald observatories did not start until after we heard of the possibility of Apollo 11 carrying the LR<sup>3</sup>, the time available for preparation was very short. Groups from Wesleyan University and the Goddard Space Flight Center worked with E. J. Wampler and other scientists from the University of California at Santa Cruz in designing, collecting, and setting up the apparatus at Lick. At the same time, groups from the University of Maryland and Goddard worked with J. E. Floyd and others from the University of Texas in designing, constructing, and putting into operation the apparatus at McDonald.

Range predictions for each site giving the range to the expected landing site at frequent intervals with the highest accuracy possible at that time were prepared at the Jet Propulsion Laboratory (JPL). With these predictions, it was possible to open a range gate in the timing electronics so that only pulses hitting the photomultiplier during a time interval of 10 or 20 microseconds around the expected time of signal return were recorded. Bin systems were used at both sites to sort the apparent signal pulses according to their time of arrival. If the number of pulses in one bin was well above the average number of pulses in other bins during the firing of a number of shots in a run, this was taken as the criterion for real signals being received.

Q-switched ruby lasers with pulse lengths of tens of nanoseconds, energies of about 7 joules per pulse, and repetition times of 3 to 30 seconds were used at both sites. Each laser beam was arranged so that it appeared to diverge from the Coudé focus of the telescope and fill the aperture. By using the telescope to reduce the divergence of the original laser beam, it was possible to produce a beam with only about 2 arc seconds angular divergence and to illuminate a small spot on the moon. Since the beam divergence was close to the limitations on astronomical seeing set by atmospheric turbulence, great care was needed to ensure that the transmitted beam was pointed at the correct spot on the lunar surface (see Fig. 2). The Coudé focus of each telescope was used because this focus remains fixed with respect to the earth even though the telescope points in different directions; thus the laser and

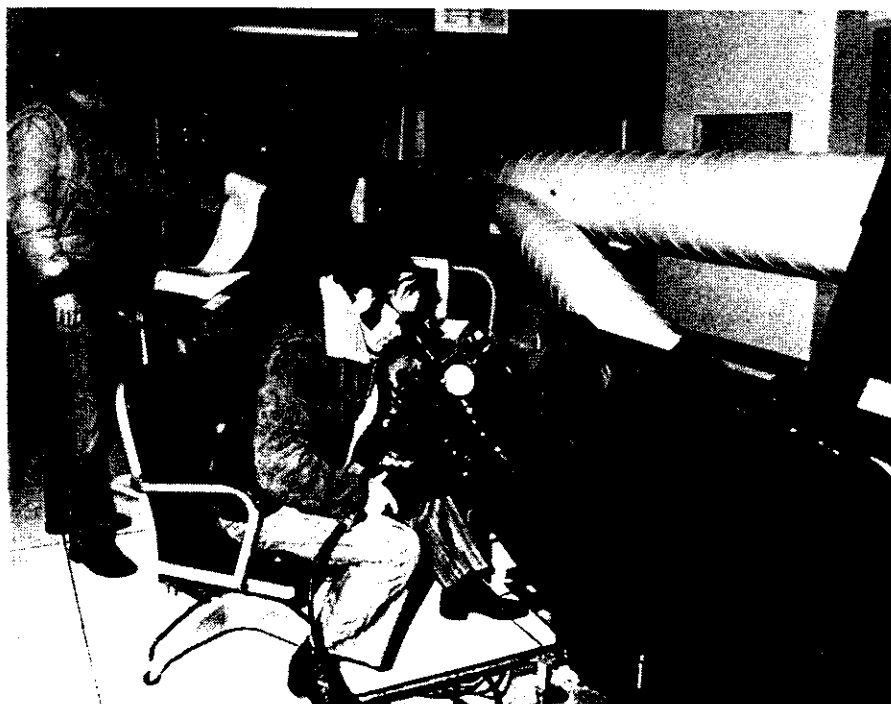


Fig. 2. The guiding station at the McDonald Observatory where the transmitted laser beam is pointed at the desired location on the lunar surface.

electronics did not have to move with the telescope.

The Apollo reflector panel was placed on the lunar surface by astronauts Armstrong and Aldrin on 21 July 1969. Attempts were made almost immediately to obtain range measurements, but these were hampered by the brief time available before the moon became too low in the sky on that night and by some initial uncertainty in the actual landing site. Ground instrument difficulties and weather problems caused some further delays, but on 1 August the Lick Observatory succeeded in obtaining strong return signals from the Apollo 11 reflector (11) and in measuring the difference between the observed and predicted range with an accuracy of 7 m (12, 13). Soon afterward, return signals were obtained with a high confidence level at McDonald also (14). Several early discussions of the experiment and the preliminary results from Lick and McDonald are available (12, 13, 15). Successful range measurements to the Apollo 11 reflector have also been reported by the Air Force Cambridge Research Laboratories Lunar Ranging Observatory in Arizona (16); the Pic du Midi Observatory in France (17, 18); and the Tokyo Astronomical Observatory in Japan (19).

#### Additional Lunar Retroreflectors

Four additional reflector panels have been placed at other locations on the lunar surface since 1969. The first was a French-built package of 14 glass corner reflectors, each 11 cm on an edge, carried to the moon by the Soviet spacecraft Luna 17 in November 1970. The package was mounted on the eight-wheeled lunar exploration vehicle Lunakhod 1. Return signals from it have been observed by a Soviet group (20) using a 2.6-m (102-inch) telescope at the Crimean Astrophysical Observatory and by a French group (21) using the 1.06-m (42-inch) telescope at the Pic du Midi Observatory. The package was not designed to give return signals during a lunar day. No observed returns have been reported since the first few months after landing, and the reflector thus may have been coated with dust stirred up during surface explorations.

The next two lunar reflector arrays were carried on the Apollo 14 and Apollo 15 missions. The retroreflectors used in both arrays were similar to those

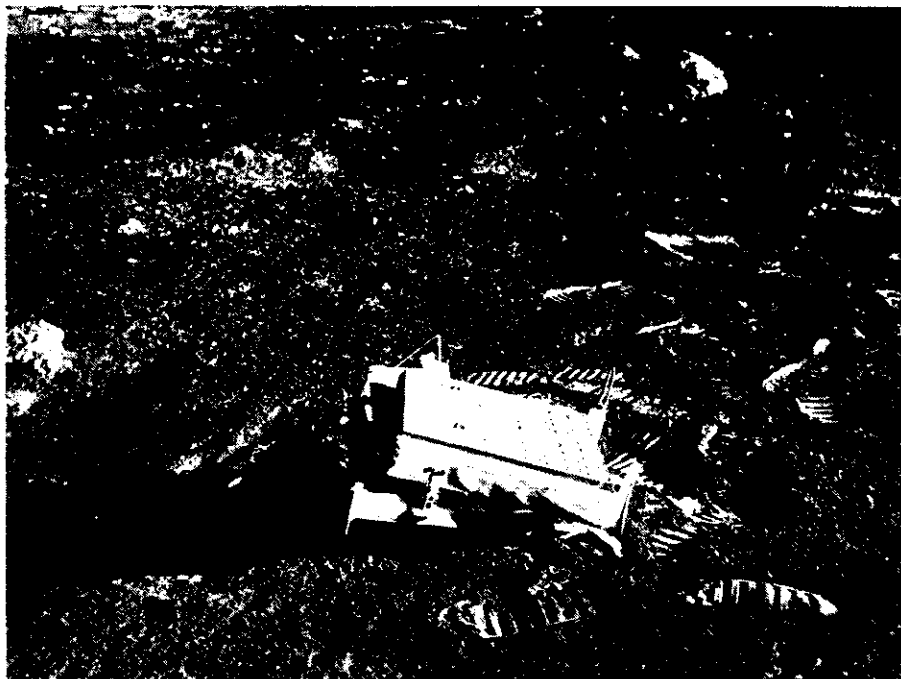


Fig. 3. Apollo 14 reflector package after installation at the Fra Mauro landing site.

employed for Apollo 11. The overall design of the Apollo 14 array (see Fig. 3) is similar to that for Apollo 11, except for some modifications of the supporting pallet to minimize weight, and the number of corner reflectors is the same. The array was deployed on 5 February 1971, and return signals were observed by the McDonald Observatory the same day.

The Apollo 15 array contains 300 corner reflectors mounted in a hexagonal close packed arrangement in order to minimize the size and weight. The overall dimensions are 104 cm by 61 cm. After deployment on 31 July 1971, returns were obtained at McDonald within a few days. A major purpose in making the array larger was to permit regular observations with simpler ground equipment for groups of investigators who are interested mainly in obtaining geophysical information, and who therefore do not have to observe more than one reflector. An additional advantage in using the Apollo 15 array is that observable lunar surface features located nearby simplify the guiding of the telescope. Returns from this array have also been obtained by groups from France (22), the Soviet Union (23), the Smithsonian Astrophysical Observatory (24), and the Air Force Cambridge Research Laboratories (25). Further information on the Apollo arrays and the corner reflectors used in them is provided elsewhere (9, 10, 26).

The fifth reflector package was re-

cently carried to the moon by Luna 21, which landed on 15 January 1973. It is a French-built package similar to that carried by Luna 17, and is mounted on Lunakhod 2. Return signals from it were obtained by the McDonald Observatory during the first and second lunar nights after landing. French scientists Orszag and Calame from the Ecole Polytechnique in Paris took part in obtaining the initial measurements, the results of which were reported in an article prepared jointly by Soviet, French, and U.S. scientists (27). No other returns from Lunakhod 2 have been reported.

The locations of the five lunar reflectors are shown in Fig. 4. The three Apollo reflectors form a large triangle on the lunar surface with sides of 1250, 1100, and 970 km. The complex angular motions of the moon about its center of mass can be separated with high accuracy from the range changes due to center-of-mass motion by differential range measurements to the different reflectors.

#### Range Measurements at McDonald

The 2.7-m telescope at the McDonald Observatory is a general-purpose instrument used in a variety of observational programs. Typically, the measurement of lunar ranges has been scheduled for three observing periods per day, weather permitting, except close to new moon. One of these ob-

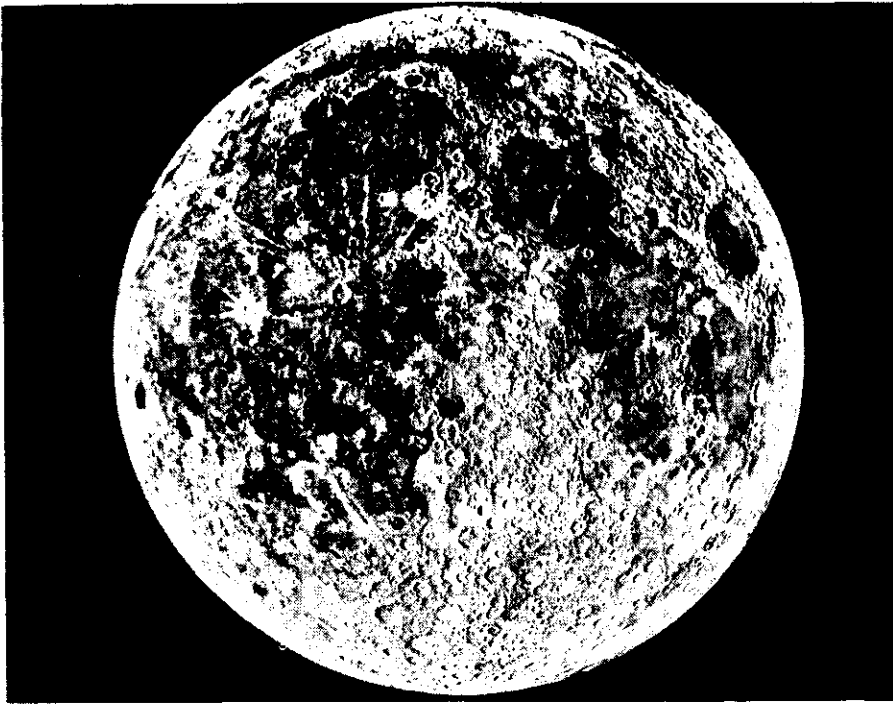


Fig. 4. Locations of the Apollo and Lunakhod reflector packages on the lunar surface.

servicing periods is chosen to be when the moon is near its highest point in the sky, and the others about 3 hours earlier and later. The total scheduled telescope time for the experiment is about 60 hours per month. The numbers of successful runs per half year, starting from the first half of 1970, have been 7, 55, 83, 160, and 226, respectively. A successful run is defined here as a sequence of perhaps 50 to 300 laser shots, fired over a period of

from 5 to 20 minutes, in which a statistically significant number of consistent return signals is obtained.

After the initial acquisitions with the electronic bin system, the electronics at McDonald were put into their originally intended operating mode. The basic system, which was designed at the University of Maryland, depends on the time delay measurements being made in two parts (28). One part consists of the integral number of 50-nanosecond

intervals being determined between clock pulses which occur during the roughly 2.4- to 2.7-second transit time of the laser pulse out to the moon and back. The other consists of highly accurate measurements being made of the time delays: (i) between a start pulse generated by the outgoing laser pulse and the first subsequent clock pulse, and (ii) between a stop pulse generated by the photomultiplier which receives the returned light collected by the telescope and the following clock pulse. The circuits used can be calibrated to an accuracy of 0.1 nsec or slightly better. The number of 50-nsec intervals and the time intervals for start and stop pulses are written down on magnetic tape after each apparent return observed by the photomultiplier, and these are later combined with circuit calibrations and accurate clock information to give the final transit time measurement.

The laser that has been in use at the McDonald Observatory since October 1969 is a four-stage Q-switched ruby laser manufactured by the Korad Corporation. After the optical energy in the first stage has built up following the Q-switch, the reflectivity of one mirror is reduced rapidly so that the stored energy is dumped in a single short pulse lasting about 4 nsec. This pulse is then amplified by the other three stages, giving an output energy of 3 joules at 6943.0 angstroms during routine use. The repetition rate is one pulse every 3 seconds, and the laser beam divergence is about  $1.2 \times 10^{-3}$  rad

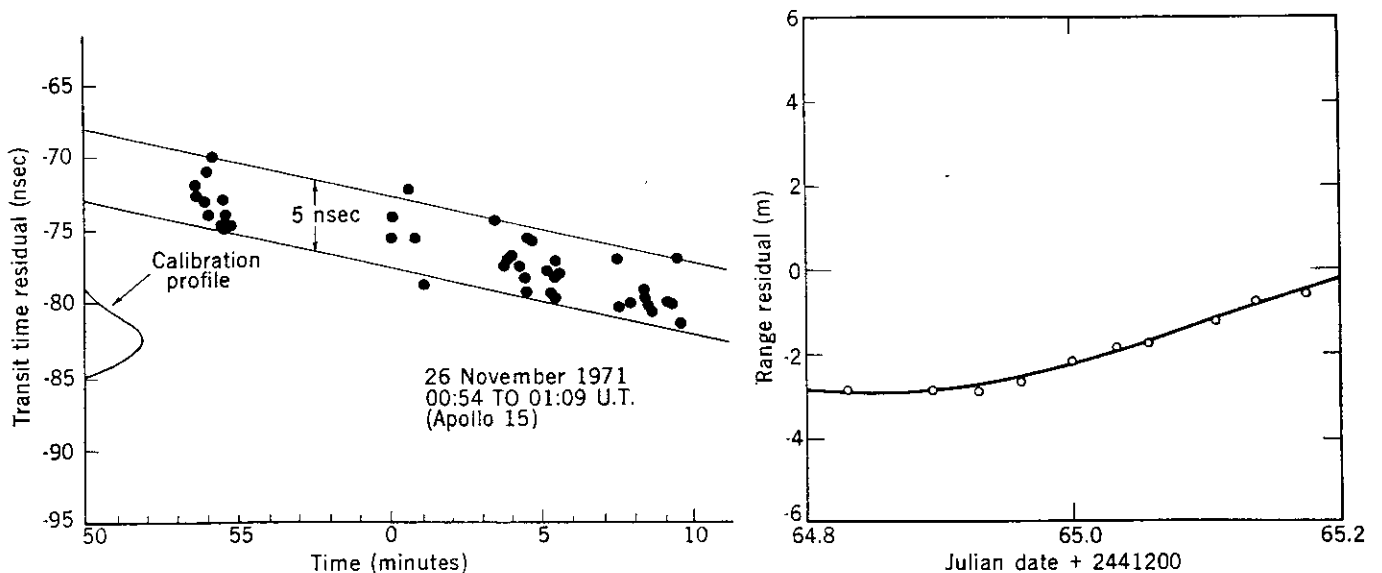


Fig. 5 (left). Individual range residuals (observed minus calculated) obtained over a 20-minute period. The scatter is consistent with that expected from the laser pulse length and the photomultiplier jitter. Fig. 6 (right). Average range residuals obtained during a series of runs covering an interval of 10 hours. The "short-arc" theoretical curve fit to the data gives 11-cm root-mean-square residuals. The time scale is in Julian days.

(angular diameter). The laser has produced over 300,000 pulses for use in lunar ranging.

Even with a 3-joule transmitted pulse and a roughly 2 arc second transmitted beam divergence, the returned optical pulse obtained with the present McDonald system is small. This is mainly due to the relatively low reflectivity in the red of the aluminum coatings on three of the mirrors in the telescope Coudé system, from each of which the light is reflected twice, and the low transmission of the narrow-band spectral filter which is used to reduce stray light. The probability of more than one photoelectron being ejected from the cathode of the photomultiplier by the returned optical pulse is small, so that the range measurements are normally made with single photoelectron pulses. Under good conditions, such signal pulses are obtained for roughly 20 percent of the laser shots fired. The statistical fluctuation in the range measurement for a single pulse is roughly  $\pm 2$  nsec because of the laser pulse length and some jitter in the photomultiplier. The statistical uncertainty can be reduced to below 1 nsec by averaging the range residuals over five or more returns.

The overall accuracy of range data taken at the McDonald Observatory between October 1969 and November 1971 was limited by uncertainties in the time delays associated with the photodetectors and the electronics. Since March 1970, the calibration uncertainty has normally been 2 nsec or better. However, on some occasions systematic errors of up to 150 nsec occurred because of triggering of the electronics by noise pulses associated with the laser firing sequence. Since December 1971, a new calibration system has been in use which usually achieves an accuracy of 0.4 nsec or better for the overall electronic time delay (29). This system is equivalent to our using range measurements to a nearby target for calibration purposes.

The range data must be analyzed carefully to discriminate between single photoelectron events caused by real returns and by stray light from the moon or the earth's atmosphere. It is necessary to assume a noise model and apply a statistical filtering technique to separate signal from noise (30). The observations identified by the filtering process may then be used individually or reduced to one compressed "normal point" per run. Both the unfiltered photoelectron events and the filtered

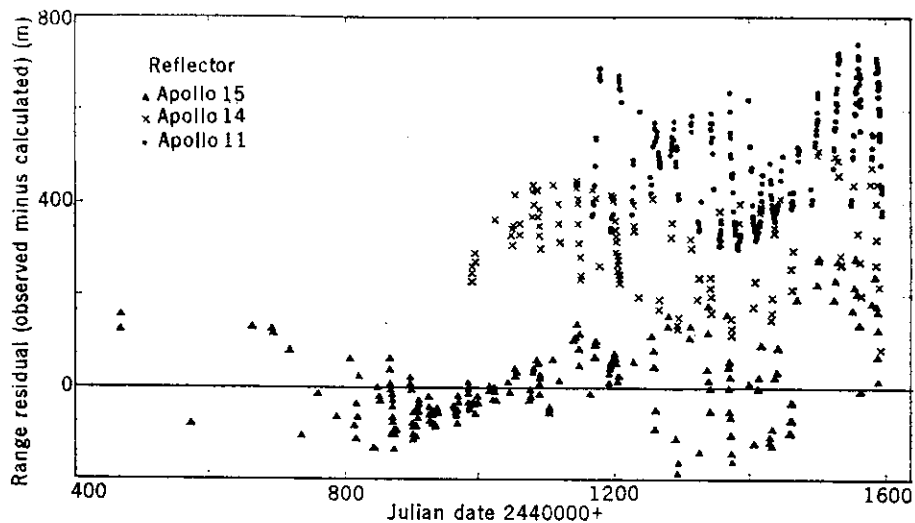


Fig. 7. Residuals obtained with the best lunar ephemeris available in 1969 and "nominal" retroreflector coordinates based on spacecraft tracking data which were available shortly after the landings. The data cover the period from August 1969 through June 1972.

observations are deposited in the National Space Science Data Center on a regular schedule, so that they are available to all interested scientists. In addition, "normal points" representing the filtered returns through 1971 are being prepared for publication (30).

The uncertainty in the averaged residual for a run is obtained by adding the statistical and electronic correction uncertainties quadratically. The usual overall accuracy obtained since December 1971 is 1 nsec, with higher accuracy being achieved on occasions when a large number of returns is observed. The uncertainties in the two other known corrections to the observed range are small enough to be neglected at present. These corrections are the atmospheric time delay and the effect of the earth tides on the station position. Polar motion and earth rotation are two of the phenomena to be studied with laser range data, so no allowance for the uncertainty due to these effects is included in the overall range uncertainty.

Examples of range data taken at the McDonald Observatory are shown in Figs. 5 and 6. The signal pulse shape shown at the left of Fig. 5 is derived from the calibration system data for the run, and thus includes the effects of photomultiplier and timing system jitter, as well as the laser pulse shape. The drift in the residuals during the run is due to inaccuracies in the right ascension and declination of the moon used in the range calculations.

Figure 6 shows the results obtained during a 10-hour period in which runs were made about every hour. The theo-

retical curve was fitted to the data by starting from our best current orbit for the moon, and then making very small corrections to the lunar hour angle, declination, and range on that day. The 0.11-m root-mean-square residuals from the theoretical curve indicate that there are no unexpected short-period effects present which cannot be modeled by adjusting known parameters in the problem.

#### New Results from Lunar Range Data

The analysis of the laser lunar range data obtained at the McDonald Observatory is still in its early stages. However, the progress made in reducing the range residuals via improvements in the range calculations is already substantial. In this section we describe the improvements that have been made and discuss the factors which limit the accuracy of the results. For the present, the angular orientation of the earth and the position of the pole are being obtained by interpolation from the smoothed 5-day mean values provided by the Bureau International de l'Heure (BIH). The atmospheric correction used is 1.88 m times the secant of the zenith angle, which is derived from the assumption of hydrostatic equilibrium for the atmosphere and the mean surface pressure at McDonald. No correction has been made yet for earth tides.

For comparison, we show in Fig. 7 the range residuals through June 1972, as calculated using the "nominal" reflector coordinates and the lunar

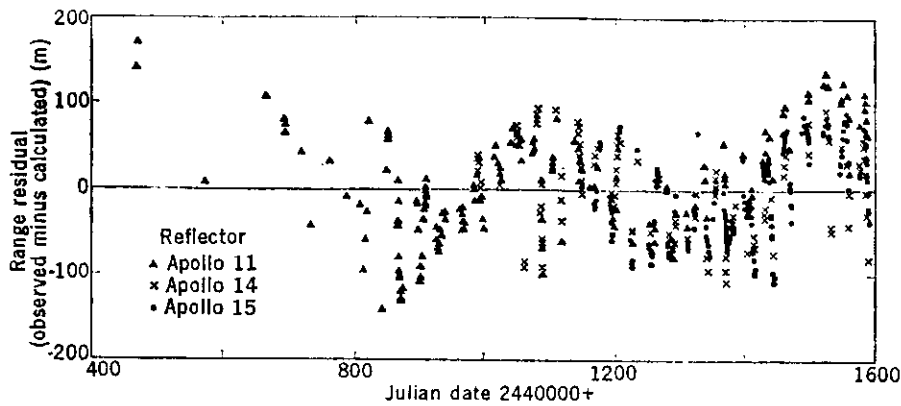


Fig. 8. Same as Fig. 9, except that the reflector coordinates and the longitude of McDonald have been adjusted for best agreement with the laser range data. The root-mean-square residuals are 54 m.

ephemeris designated LE-16 (31). The "nominal" coordinates were obtained by analysis of spacecraft tracking data near the time of the landings. LE-16 is the ephemeris (that is, the listing of lunar coordinates with respect to the earth as a function of time) which was used in the early range predictions (13). The points plotted in Fig. 7 and those in Figs. 8 and 9 are "normal points." The total uncertainty is typically 0.30 to 0.45 m for the points from February 1970 to November 1971, and is usually 0.15 m after that.

The basic quantity measured in the lunar ranging experiment is the optical transit time to the reflectors and back. However, for easier visualization of the results, it is sometimes desirable to express them in length units. For this purpose we use the value of the speed of light as adopted by the International Astronomical Union:  $c = 299,792,500$  m/sec. The uncertainty in the measured value of the speed of light does not affect the scientific results in any way that we are aware of, as discussed elsewhere (32). However, if preferred, the uncertainty in the range when quoted in meters can be regarded simply as a

conversion of the uncertainty in travel time to distance units.

The residuals of Fig. 8 are obtained if one solves for the values of the retroreflector coordinates and of the geocentric longitude at the McDonald Observatory by least squares adjustment. The root-mean-square value of the residuals is reduced to 54 m, compared to 381 m for the case shown in Fig. 7. The height of the McDonald Observatory above the equatorial plane was obtained by combining the SAO (Smithsonian Astrophysical Observatory) 1969 Standard Earth (33) coordinates for the satellite tracking station at Organ Pass with high-precision geodetic survey data connecting McDonald to Organ Pass (34). The spin-axis distance obtained at McDonald was derived from lunar range observations when the moon was near zero declination (35), and agrees well with the value obtained at Organ Pass (33, 34).

The physical librations of the moon (that is, deviations from uniform rotation due to torques acting on the moon) must be included in calculating the range. The libration theory employed in obtaining Figs. 7 and 8 corresponds

to Eckhardt's results for the second degree terms in the lunar mass distribution (36). The values of the libration parameters  $\beta$  and  $\gamma$  were taken to be 0.000627 and 0.000230, corresponding to the parameters used by Koziel (37) of  $I = 5524$  arc second and  $f \approx 1 - (\gamma/\beta) = 0.633$ , where  $I$  is the inclination of the mean lunar rotation axis to the ecliptic. Here the libration parameters  $\beta$  and  $\gamma$  are defined as  $\beta = (C - A)/B$  and  $\gamma = (B - A)/C$ , where  $A$ ,  $B$ , and  $C$  are the lunar moments of inertia about the three principal axes. If the libration parameters are adjusted simultaneously with the reflector coordinates and the geocentric longitude at the McDonald Observatory, the root-mean-square residuals are reduced to 48 m, which we believe is a reasonable measure of the range accuracy obtainable with the LE-16 ephemeris.

The ephemeris discussed above was constructed in 1968-1969 by means of a mixture of numerical and analytic techniques, as described in Garthwaite *et al.* (31). These techniques served only to remove certain numerical defects in the Brown theory, so the perturbing solar and planetary data were chosen as consistently as possible with that theory. Thus, the ephemeris was known to contain model errors due to inadequate data on solar and planetary positions and masses, which errors however were expected to be smaller and of longer period than those removed in the construction process.

Improvement of the lunar ephemeris based directly on observations was begun in 1970 with a numerical integration of the moon and major planets (38), the planetary masses and planetary starting conditions used being appropriate to the ephemeris DE-69 obtained at the JPL (39). This ephemeris was differentially corrected to fit the U.S. Naval Observatory 15-cm (6-inch) transit circle observations of the moon from 1950 to 1968.

By mid-1971, enough laser ranging data were available for us to attempt basing an orbit correction on them. Consequently, these early ranges were used to derive corrections to the lunar eccentricity, the mean longitude of perigee, and the mean longitude of the lunar center of mass. Starting conditions based on these corrections were used for new integrations at both the JPL and the University of Texas at Austin. The resulting ephemerides and the corresponding parameter set have been discussed briefly elsewhere (40), but it should be emphasized that

Table 1. Parameters consistent with the LURE-1 lunar ephemeris and current libration theory. The symbols  $r$ ,  $\lambda$ , and  $\phi$  are the selenocentric reflector coordinates with respect to the "mean earth direction and mean rotation axis" coordinate system;  $\lambda'$  and  $\phi'$  are the selenocentric principal axis coordinates.  $\lambda(\text{MCD})$ ,  $\sigma(\text{MCD})$ , and  $z(\text{MCD})$  are, respectively, the McDonald Observatory geocentric longitude, spin-axis distance, and height above the equatorial plane, with respect to the Conventional International Origin.  $\beta$  and  $\gamma$  are the lunar libration parameters.

Parameters	Apollo 11	Apollo 14	Apollo 15
$r(\text{km})$	1735.647	1736.499	1735.593
$\lambda(^{\circ})$	23.4711	- 17.4773	3.6196
$\phi(^{\circ})$	0.6729	- 3.6457	26.1291
$\lambda'(^{\circ})$	23.4008	- 17.5473	3.5499
$\phi'(^{\circ})$	0.6920	- 3.6258	26.1500
$\lambda(^{\circ})(\text{MCD}) = 255.97805$		$\beta = 630.65 \times 10^{-6}$	
$\sigma(\text{MCD}) = 5492.416 \text{ km}$		$\gamma = 226.0 \times 10^{-6}$	
$z(\text{MCD}) = 3235.694 \text{ km}$			

they were intended only for interim use and should not be employed for other purposes.

Recently, additional experimental lunar ephemerides have been generated at the University of Texas (38) and at JPL (41). One of these, which we will call LURE-1, will be discussed in the remainder of this article. The same planetary starting conditions, planetary masses, and lunar mass were used as for DE-69. The mean motion of the moon was adjusted, as well as the eccentricity constant, the mean longitude of perigee, and the mean longitude of the center of mass. The range data obtained at the McDonald Observatory over a roughly 3-year period through June 1972 were used. The secular acceleration in the longitude of the moon by tidal dissipation in the earth was not included, and the integration of the ephemeris was limited to a 5.5-year period (28 June 1969 to 19 December 1974). A model error existed in the way the figure of the earth was included, but this probably did not affect the accuracy with which the data could be fitted because of the relatively short time span involved.

It is necessary to note that an unknown model error may yet exist in these integrations. It was noted by Oesterwinter and Cohen (42) that a lunar longitude discrepancy of some 15 arc seconds per century occurred when their integrator and ours were exercised with the same initial conditions. Subsequent investigation has reduced this figure to 11 arc seconds per century, and tests are continuing, but at present the source of the discrepancy is unknown. Since we fit observations, the effect on the present work is probably small, even if the flaw is in our program.

The improved physical libration theory used in our recent work has been described by Williams *et al.* (41). A number of long period terms based on additive and planetary terms listed in the *Improved Lunar Ephemeris 1952-1959* (43) have been added to Eckhardt's libration series (36, 44), including two small planetary perturbations which are nearly resonant with the moon's roughly 3-year resonance period for free librations in longitude. The effect of third and fourth degree terms in the lunar mass distribution, which have been discussed recently by Eckhardt (45) and by Kaula and Baxa (46), were included.

The accuracy of the LURE-1 ephemeris is still uncertain because of the limitations of even the improved libra-

Table 2. Changes in parameters obtained for two cases where the libration model was altered (see text). Numbers in parentheses indicate Apollo landing sites.

Parameter change	Model A	Model B
$\Delta\beta$	$-1.0 \times 10^{-6}$	$+0.1 \times 10^{-6}$
$\Delta\gamma$	$-2.1 \times 10^{-6}$	$+2.3 \times 10^{-6}$
$\Delta\lambda(\text{MCD})$	- 1 m	- 2 m
$\Delta r(11)$	- 6 m	0 m
$\Delta r(14)$	- 4 m	+ 4 m
$\Delta r(15)$	- 50 m	- 10 m
$\Delta\lambda(11)$	- 20 m	+ 40 m
$\Delta\lambda(14)$	+ 20 m	+ 40 m
$\Delta\lambda(15)$	+ 100 m	+ 230 m
$\Delta\phi(11)$	- 45 m	- 20 m
$\Delta\phi(14)$	+ 50 m	+ 10 m
$\Delta\phi(15)$	- 120 m	- 50 m

tion theory (41). The same is true for the corresponding selenocentric reflector coordinates, libration parameters, and the geocentric longitude of the McDonald Observatory. The values of these quantities, as obtained from a least squares solution with the LURE-1 ephemeris, are given in Table 1. It is interesting that the geocentric longitude of McDonald is 0.9 arc second higher than the value obtained from the SAO 1969 Standard Earth coordinates for the satellite tracking station at Organ Pass (33, 34). This difference is similar to the roughly 0.8 arc second difference between longitudes determined by the Deep Space Tracking Network stations of the JPL and by the SAO (33). Our current values for the other two McDonald coordinates, which were obtained as discussed earlier, are

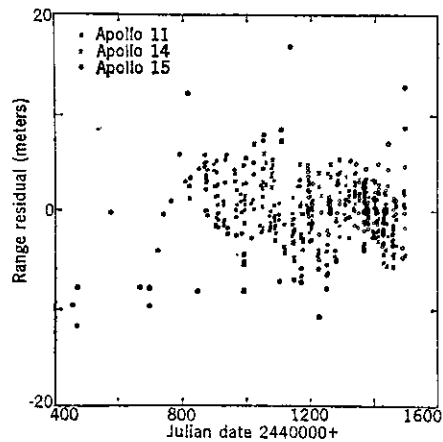


Fig. 9. Range residuals obtained after adjusting the reflector coordinates, four parameters for the lunar orbit, the differences in moments of inertia of the moon about its principal axes, the longitude of McDonald, and five of the third degree harmonic coefficients in the lunar mass distribution. The overall root-mean-square residuals are 3 m.

also included in Table 1. The resulting residuals have a 5-m root-mean-square amplitude.

Some of the third degree lunar harmonics cause large fixed offsets in the principal axes with respect to the mean direction of the earth and the mean axis of rotation. However, it is the coordinates of the reflectors with respect to the mean direction of the earth and the mean axis of rotation which are determined most directly from the lunar ranging data. The uncertainties in the third degree harmonic coefficients are sufficiently large that one can obtain the "mean earth plus rotation axis" coordinates more accurately than the principal axis coordinates. Both sets of coordinates are given in Table 1, but it should be remembered that the principal axis coordinates may have much larger uncertainties than the other ones.

In order to show how sensitive the reflector coordinates and libration parameters are to the libration model used, we give in Table 2 the changes in these quantities from those given in Table 1 for two different models. Model A corresponds to no third degree terms in the librations, while model B corresponds to our adjusting all of the third degree harmonic coefficients except  $C_{30}$  and  $S_{31}$  in order to minimize the root-mean-square residuals. The resulting residuals for Model B are plotted in Fig. 9, and the root-mean-square residuals are 3 m. These harmonic coefficients,  $C_{30}$  and  $S_{31}$ , were not adjusted because moderate variations in them do not significantly affect the range residuals. The derived corrections to the other third degree harmonic coefficients cannot be regarded as meaningful at present, since the currently used libration theory does not give the correct orientation of the moon within better than an arc second even if exactly correct values of the libration parameters and third degree coefficients are used. However, attempts are being made at several laboratories to make major improvements in the solution of the libration equations (41) and it may become possible to determine several of the third degree terms in the lunar mass distribution from the lunar range data.

The variations in the reflector coordinates given in Table 2 are 50 m or less for Apollo 11 and Apollo 14, but up to 230 m for one coordinate of the Apollo 15 reflector. These can perhaps be taken as representative of the present uncertainty in the coordinate differences



between the three reflectors. However, it should be remembered that the quantity  $G(M_{\text{earth}} + M_{\text{moon}})$ , where  $G$  is the gravitational constant and  $M$  is mass, has not been varied in the solutions discussed here, and as a result the error in the semimajor axis of the lunar orbit may be masked by corresponding errors in the rectangular coordinates of all three reflectors in the mean direction of the earth. This might amount to several hundred meters. Common errors nearly as large in the other two coordinates of all three reflectors are also possible because of limitations in the LURE-1 ephemeris. Furthermore, the origin of longitudes in LURE-1 is determined by the orbit of the earth as represented in the DE-69 planets, not by the lunar ranging data. Thus, although the present results should be accurate in terms of variations in the lunar distance or differences in coordinates of the reflectors, further analysis is required before the angular position of the moon and the absolute coordinates of the reflectors can be obtained reliably.

A value for the normalized lunar moment of inertia  $C/MR^2$ , where  $R$  is the radius of the moon, based on the lunar range data discussed above has been derived recently (41). The values of the libration parameters  $\beta$  and  $\gamma$  which were used are close to those given in Table 1, and the uncertainties were taken to be  $0.5 \times 10^{-6}$  and  $3 \times 10^{-6}$ , respectively. These results were combined with a value of  $C_{20} = -204 (+4, -1) \times 10^{-6}$  ( $C_{20}$  being the harmonic coefficient) obtained from Lunar Orbiter tracking data to give  $C/MR^2 = 0.395 (+0.006, -0.010)$ . This value is consistent with a nearly uniform density distribution for the moon.

### Future Prospects

When the solution of the libration equations to high accuracy has been completed, we expect to be able to fit the lunar range data within the combined uncertainty of the measurements and the BIH corrections for polar motion and the rotational position of the earth. The 5-day mean values obtained by the BIH are currently believed to be accurate to  $\pm 40$  cm for each component of polar motion and  $\pm 0.03$  arc second in angular position (47). The error in range due to uncertainty of the earth's angular position can be as high as 70 cm well before or after meridian passage, which is considerably larger

than the uncertainty of the present range measurements. However, on days when measurements before, during, and after meridian passage are available, the corrections necessary for polar motion and earth rotation can be obtained directly from the range data. Measurements on such days, or when the moon is near the zenith, are expected to give the most reliable data for determining the lunar orbit and other information. The entire set of data can then be re-analyzed to obtain information on polar motion and earth rotation. The data obtained at the McDonald Observatory from December 1971 on, which have an accuracy of  $\pm 15$  cm, will be particularly useful for this purpose. However, data from additional stations will be needed before the results can be anywhere near complete.

A new lunar ranging station (48) is being built on the island of Maui in Hawaii under the direction of the Institute for Astronomy of the University of Hawaii. It is expected to be completed in 1973, and a regular observing program will be started as soon as possible. The station has been designed with a particular goal in mind—that of achieving an ultimate accuracy of 2 to 3 cm for the "normal point" representing the results of a typical run. However, some time undoubtedly will be needed before this goal can be achieved.

In the future it is hoped that lunar ranging stations in other countries will be able to participate in observation programs in which at least one lunar reflector is observed three times daily whenever observing conditions permit. The participation of about 6 to 12 stations located in good sites throughout the world would enable us to obtain regular measurements of polar motion changes and fluctuations in the earth's rotation rate which may occur at intervals of down to 1 day. Such measurements, together with measurements obtained by long baseline microwave interferometry (49) and laser range measurements to high altitude satellites (50), would greatly improve our knowledge of polar motion and earth rotation.

Lunar ranging stations now exist at the Crimean Observatory in the U.S.S.R. and at the Pic du Midi Observatory in France. By 1975 it is expected that the French station will have been moved to a new location near Grasse where a program of more frequent observation will be possible (51). The Air Force Cambridge Research Laboratories

lunar laser ranging system, which was in Arizona until 1972, has been moved to Australia. With the participation of the Smithsonian Astrophysical Observatory, this system will be operated near Canberra by the Division of National Mapping of the Australian Department of Minerals and Energy. Partial support for a new lunar ranging station in Japan has been obtained recently (52). A joint British-South African station in South Africa has been proposed (53), and interest in lunar ranging has been expressed by groups in several other countries. We hope that all these efforts will lead to the establishment of a permanent international service for monitoring polar motion and fluctuations in the earth's rotation.

One of the phenomena of particular interest in connection with polar motion is a wobble of the figure axis of the earth around the rotation axis with a period of roughly 14 months. This "Chandler wobble" (54) would die down with time and the figure axis would line up with the rotation axis if there were not something that frequently reexcited it. Smylie and Mansinha (55) have suggested that the excitation is caused by major earthquakes in which enough mass is shifted to change the earth's inertial properties significantly. However, other authors have raised substantial questions about this (56, 57), and some recent calculations based on current understanding of the actual ground motions taking place during earthquakes do not support this idea (57). On the other hand, it is not clear at present that any other known process going on in the earth, the oceans, or the atmosphere can explain the observed amplitude of roughly 2 to 10 m for the Chandler wobble. Turbulent eddies in the core seem to be another possibility, but there are difficulties in obtaining sufficient coupling of core motions to the inner part of the mantle (58). Until this problem is resolved, there is a chance that some aspect of our current understanding of earthquakes is in error. Improved measurements of polar motion from lunar ranging and other techniques are very much needed. The expected accuracy of lunar ranging for determining polar motion and earth rotation (48, 59), as well as for obtaining improved information on station locations, the lunar mass distribution, retroreflector locations, and the lunar orbit (18, 60, 61), has been discussed elsewhere.

In addition to the above types of information, we soon hope to use the

lunar range data to carry out a check on gravitational theory. Nordvedt (62) has pointed out that in some gravitational theories the ratio of the gravitational mass to the inertial mass of large bodies can be slightly different from unity. This is because the gravitational self-energy contribution to the mass of the body behaves differently from the rest of the mass. It appears that the lunar range data will soon provide a strong indication of whether gravitational and inertial mass are the same for terrestrial-sized bodies (61). For the scalar-tensor theory of gravitation, for example, with a value of the coupling constant consistent with the solar-oblateness observations ( $\omega = 5$ ), the moon's orbit should be displaced toward the sun by approximately 120 m (63).

## Summary

The lunar ranging measurements now being made at the McDonald Observatory have an accuracy of 1 nsec in round-trip travel time. This corresponds to 15 cm in the one-way distance. The use of lasers with pulse-lengths of less than 1 nsec is expected to give an accuracy of 2 to 3 cm in the next few years. A new station is under construction in Hawaii, and additional stations in other countries are either in operation or under development. It is hoped that these stations will form the basis for a worldwide network to determine polar motion and earth rotation on a regular basis, and will assist in providing information about movement of the tectonic plates making up the earth's surface. Several mobile lunar ranging stations with telescopes having diameters of 1.0 m or less could, in the future, greatly extend the information obtainable about motions within and between the tectonic plates.

The data obtained so far by the McDonald Observatory have been used to generate a new lunar ephemeris based on direct numerical integration of the equations of motion for the moon and planets. With this ephemeris, the range to the three Apollo retroreflectors can be fit to an accuracy of 5 m by adjusting the differences in moments of inertia of the moon about its principal axes, the selenocentric coordinates of the reflectors, and the McDonald longitude. The accuracy of fitting the results is limited currently by errors of the order of an arc second in the angular orientation of the moon, as derived from the best available theory

of how the moon rotates in response to the torques acting on it. Both a new calculation of the moon's orientation as a function of time based on direct numerical integration of the torque equations and a new analytic theory of the moon's orientation are expected to be available soon, and to improve considerably the accuracy of fitting the data.

The accuracy already achieved routinely in lunar laser ranging represents a hundredfold improvement over any previously available knowledge of the distance to points on the lunar surface. Already, extremely complex structure has been observed in the lunar rotation and significant improvement has been achieved in our knowledge of lunar orbit. The selenocentric coordinates of the retroreflectors give improved reference points for use in lunar mapping, and new information on the lunar mass distribution has been obtained. Beyond the applications discussed in this article, however, the history of science shows many cases of previously unknown phenomena discovered as a consequence of major improvements in the accuracy of measurements. It will be interesting to see whether this once again proves the case as we acquire an extended series of lunar distance observations with decimetric and then centimetric accuracy.

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