CONTRIBUTIONS TO THE THEORY*
OF ATMOSPHERIC REFRACTION

Part II. Refraction Corrections in Satellite Geodesy

A. Photogrammetric Refraction

Depending on its definition, satellite photogrammetry deals with vertical photography of the earth's surface taken from an orbiting satellite, or with photography of an orbiting satellite taken from the surface of the earth against the stellar background. The photogrammetric refraction is essentially the same in both cases.

To determine the geometric relationship between astronomical refraction \( \Delta z_1 \) and photogrammetric refraction \( \Delta \theta_2 \), we have from Fig. 3

\[
\frac{\overline{OQ}}{\overline{P_1 Q}} = \frac{\sin z_2}{\sin z_2}
\]

and

\[
d = (r_2 - \overline{OQ}) \sin z_2 = r_2 \sin z_2 - r_1 \sin (z_1 + \Delta z_1)
\]

\[
s = \overline{P_1 Q} + d \cot g z_2 = r_2 \cos z_2 - r_1 \cos (z_1 + \Delta z_1)
\]

The photogrammetric refraction is therefore obtained from the astronomical refraction by the formulas

\[
\begin{align*}
\Delta \theta_2 &= \frac{d}{s} = \frac{r_2 \sin z_2 - r_1 \sin (z_1 + \Delta z_1)}{r_2 \cos z_2 - r_1 \cos (z_1 + \Delta z_1)} \\
&= \frac{r_2 \sin z_2}{r_2 \cos z_2} (1 - \cos \Delta z_1)
\end{align*}
\]

applicable to both cases of satellite photogrammetry defined above.

* -- Suite et fin de l'article publié dans les 50e 105 et 106 du Bulletin Géodésique.
### Satellite and Stellar Background of White Sun

#### Differential Refraction in Micrometer Between

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>0</th>
<th>200</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
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<tbody>
<tr>
<td>Aperture Angle</td>
<td>0°</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

#### Table V:

Local length of the camera (in meters)

![Diagram](image)

**Theory of Atmospheric Refraction**

\[
\left( \frac{1}{d} \right)^2 \frac{1}{1} \left[ - \frac{1}{d} \frac{1}{3.5 \cos \theta} \left( \frac{1000}{d} \right) \frac{1}{z} \text{tan} \theta \right] = d = 0.002317
\]

Assuming an effective wavelength of 0.554 μm, the correction term is expressed in radians. Numerically,

\[
\left( \frac{1}{d} \right)^2 \frac{1}{1} \left[ - \frac{1}{d} \frac{1}{3.5 \cos \theta} \left( \frac{1000}{d} \right) \frac{1}{z} \text{tan} \theta \right] = d
\]

which gives

\[
\left( \frac{1}{d} \right)^2 \frac{1}{1} \left[ - \frac{1}{d} \frac{1}{3.5 \cos \theta} \left( \frac{1000}{d} \right) \frac{1}{z} \text{tan} \theta \right] = d
\]

For departure, we have from (36)

\[
\left( \frac{1}{d} \right)^2 \frac{1}{1} \left[ - \frac{1}{d} \frac{1}{3.5 \cos \theta} \left( \frac{1000}{d} \right) \frac{1}{z} \text{tan} \theta \right] = d
\]
J. Saastamoinen

Table VII gives the resulting standard values of $\Delta \theta_2$ for super-wide angle photography from different camera heights. Due to these refraction angles, the photographic image points will appear to be shifted radially away from the principal point of the photograph; the linear displacements (in microns) are equal to the tabular values multiplied by $f \sec^2 \theta$, $f$ denoting the focal length of the camera (in metres).

**Table VII.**

Photogrammetric Refraction in Microradians
in Vertical Photography from Satellites

<table>
<thead>
<tr>
<th>Apparent Nadir Distance</th>
<th>In Vertical Photography from Satellites</th>
<th>Orbital Height</th>
</tr>
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<tr>
<td></td>
<td>250 km</td>
<td>500 km</td>
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<td></td>
<td>750 km</td>
<td>1000 km</td>
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<td>1500 km</td>
<td>2000 km</td>
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<td>0.6</td>
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<td>20°</td>
<td>3.6</td>
<td>1.9</td>
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<tr>
<td></td>
<td>1.3</td>
<td>1.0</td>
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<tr>
<td>30°</td>
<td>5.8</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>1.7</td>
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<td>40°</td>
<td>8.6</td>
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<td>3.4</td>
<td>2.8</td>
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<td>50°</td>
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<td>59°</td>
<td>19.3</td>
<td>13.1</td>
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<td></td>
<td>14.3</td>
<td>38.6</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In photogrammetric mapping from low-altitude satellites, corrections for refraction will be required if horizontal control is extended by photogrammetric means. They can be obtained more conveniently using the approximate formulas

$$\begin{align*}
\Delta^2 &= \left( \frac{r}{r+h} \right)^2 \sin^2 \theta \\
\Delta \theta &= \frac{2.32 p \times \sin \theta}{(r+h)^2 \Delta^2 (\cos \theta - \Delta)}
\end{align*}$$

where $\Delta \theta$ is the photogrammetric refraction in microradians at apparent nadir distance $\theta$, $p$ is the barometric pressure in millibars at the ground level, $h$ is the height of the camera in kilometres above the ground level ($h > 50$ km), and $r$ is the radius of the earth in kilometres.

**Theory of Atmospheric Refraction**

B. — The Atmospheric Corrections for Troposphere and Stratosphere in Electromagnetic Ranging of Satellites

Due to the retarding effect of the atmosphere on the propagation of electromagnetic waves, a range correction

$$\Delta S = \int_{\gamma r}^{r} \frac{(n-1)}{S} dS$$

where $n$ is the refractive index referred to the group velocity of propagation, must be subtracted from observed electromagnetic distance $S$ to obtain the true measured length of the effective ray path. As far as the electrically nonconducting lower atmosphere, the troposphere and the stratosphere, is concerned, this correction can be derived on the basis of the refraction theory developed previously in Part I for the determination of astronomical refraction.

**Derivation of General Formula for Range Correction**

In a spherically layered atmosphere, the basic mathematical expression of the range correction becomes

$$\Delta S = \int_{\gamma r}^{r} \frac{(n-1) \sec z}{r} \, dz$$

which corresponds to equation (1) of astronomical refraction. Setting $n r / (n_1 r_1) = y$ for brevity, we now have from (2)

$$\sin^2 z = (\sec^2 z_1 - 1) / (y^2 \sec^2 z_1)$$

$$\cos^2 z = (y^2 \sec^2 z_1 - \sec^2 z_1 + 1) / (y^2 \sec^2 z_1)$$

and

$$\sec z = y \sec z_1 \left[ 1 + \sec^2 z_1 (y^2 - 1) \right]^{-1} = y \sec z_1 - \frac{1}{2} y (y^2 - 1) \sec^3 z_1 + \cdots$$

Neglecting the subsequent terms in the binomial expansion, the first four may be written identically

$$\frac{3}{8} y (y^2 - 1)^2 \sec^6 z_1 - \frac{5}{16} y (y^2 - 1)^3 \sec^7 z_1 + \frac{35}{128} y (y^2 - 1)^4 \sec^9 z_1 - \cdots$$
The values of the integrals on the right side have been determined

\[
\int_0^1 \left( \frac{1 + b}{1} \right) u \left( \frac{1}{1 + \frac{1}{1 + b}} \right) \left( 1 + \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = -\int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 - b \right)
\]

and integrating by parts, we obtain for these integrals

\[
dp b \left( \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = dp \\
\int \frac{1}{1 - u} du = \int \\
(1 + b) \left( \frac{1}{1 - b} \right) = \lambda
\]

Using the substitution

\[
\int_0^1 \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = -\int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 - b \right)
\]

The first term in equation (5) is given by the fact that integral

\[
-\int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = \int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 - b \right)
\]

The second term breaks down into a linear function of the atmospheric integrals. By the substitution of the (π/2) integrals

\[
1 + b \left( \frac{1}{1 - b} \right) = \lambda
\]

The second term in equation (5) is given by the fact that integral

\[
\int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = \int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 - b \right)
\]

The second term breaks down into a linear function of the atmospheric integrals. By the substitution of the (π/2) integrals

\[
1 + b \left( \frac{1}{1 - b} \right) = \lambda
\]

The theory of atmospheric refraction

\[
\int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 + \frac{1}{1 - b} \right) = \int \frac{\sqrt{1 - u^2}}{1 + u} \left( \frac{1}{1 - b} \right) \left( 1 - b \right)
\]

The second term breaks down into a linear function of the atmospheric integrals. By the substitution of the (π/2) integrals

\[
1 + b \left( \frac{1}{1 - b} \right) = \lambda
\]
THEORY OF ATMOSPHERIC REFRACTION

\[
\int_{r_0}^{r} (n-1)^2 (r-r^0) \, dr = \frac{(n^0-1)^2}{4m^2} = \frac{R^2}{4g^2} (n^0-1)^2 \, T^0^2
\]

Consequently, using the identity \( r - r_1 = r^0 - r_1 + r - r^0 \) and applying integral (22), the total stratospheric component is obtained as

\[
\int_{r_0}^{r} (n-1)^2 (r-r_1) \, dr = \frac{R^2}{4g^2} (n^0-1)^2 \, T^0^2 + \frac{R}{2g} (r^0 - r_1) (n^0-1)^2 \, T^0
\]

For the tropospheric component of the same integral we have from (15) and (17)

\[
\int (n-1)^2 (r-r_1) \, dr = \frac{(n_1-1)^2 T_1}{\beta^2} \int \left( \frac{T}{T_1} \right)^{2m'} \left( \frac{T}{T_1} - 1 \right) \, dT =
\]

\[
= \frac{(n_1-1)^2 T_1}{\beta^2} \left[ \left( \frac{1}{2m'+1} \right) \left( \frac{T}{T_1} \right)^{2m'+1} - \left( \frac{1}{2m'+1} \right) \left( \frac{T}{T_1} \right)^{12m'+1} \right] + C =
\]

\[
= \frac{(n-1)^2 T_1}{\beta^2} \left[ \left( \frac{1}{2m'+1} \right) \frac{T}{T_1} - \left( \frac{1}{2m'+1} \right) \right] + C =
\]

\[
= \frac{(n-1)^2 T_1}{\beta^2 (2m'+1)} \left[ \frac{2m'+1}{2m'+2} \frac{T}{T_1} - 1 \right] + C = \frac{(n-1)^2 T_1}{\beta^2 (2m'+1)} \left[ \frac{T}{T_1} - 1 - \left( \frac{1}{2m'+2} \right) \frac{T}{T_1} \right] + C =
\]

\[
= \frac{(n-1)^2 (r-r_1) T}{\beta (2m'+1)} - \frac{(n-1)^2 T^2}{\beta^2 (2m'+1)} + C =
\]

\[
= \frac{R}{2g + R} (r-r_1) (n-1)^2 T - \frac{R^2}{2g} (2g + R) (n-1)^2 T^2 + C
\]

The tropospheric component is accordingly

\[
\int_{r_1}^{r_0} (n-1)^2 (r-r_1) \, dr = \frac{R^2}{2g} \left[ \left( n_1 - 1 \right)^2 T_1^2 - (n^0-1)^2 \, T^0^2 \right] - \frac{R}{2g + R} (r^0 - r_1) (n^0-1)^2 \, T^0
\]

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\[
\int_{r_1}^{r_1} (n-1)(n_1-n) \, dr.
\]

Since identically

\[(n-1)(n_1-n) = (n_1-1)(n-1) - (n-1)^2\]

we obtain immediately, applying integrals (9), (22) and (23)

\[
\int_{r_1}^{r_1} (n-1)(n_1-n) \, dr = \frac{R}{g} (n_1-1)^2 T_1 - \left( \frac{R}{2g + R}\right) \left[ (n_1-1)^2 T_1 + \frac{1}{2} (R\beta/g) (n^0-1)^2 T^0 \right]
\]

\[
\text{(51)}
\]

\[
\text{Integral } \int_{r_1}^{r_1} (n-1)(n_1-n)(r-r_1) \, dr.
\]

Using the same identity as in the previous case, we have first

\[
\int_{r_1}^{r_1} (n-1)(n_1-n)(r-r_1) \, dr = (n_1-1) \int_{r_1}^{r_1} (n-1)(r-r_1) \, dr - \int_{r_1}^{r_1} (n-1)^2 (r-r_1) \, dr
\]

For the stratospheric component of the second integral above, we have from (11)

\[
\int (n-1)^2 (r-r^0) \, dr = (n^0-1)^2 \int (r-r^0) e^{2m} (r-r^0) \, dr =
\]

\[
= (n^0-1)^2 e^{2m} \frac{(r-r^0)}{4m^2} \left[ 2m(r-r^0) - 1 \right] + C =
\]

\[
= (n-1)^2 \left[ \frac{1}{2m} (r-r^0) - \frac{1}{4m^2} \right] + C
\]

and
\[ \left[ \varepsilon_0 \frac{1}{1} \left( \frac{\varepsilon^1}{\varepsilon} \right) + \frac{\varepsilon^1}{\varepsilon} \right]_{12} \sec \frac{\varepsilon^1}{\varepsilon} = \frac{8\gamma}{\varepsilon^1} \]

\[ + \left[ \varepsilon_0 \frac{1}{1} \left( \frac{\varepsilon^1}{\varepsilon} \right) + \frac{\varepsilon^1}{\varepsilon} \right]_{12} \sec \frac{\varepsilon^1}{\varepsilon} = \frac{8\gamma}{\varepsilon^1} \]

\[ + \left[ \varepsilon_0 \frac{1}{1} \left( \frac{\varepsilon^1}{\varepsilon} \right) + \frac{\varepsilon^1}{\varepsilon} \right]_{12} \sec \frac{\varepsilon^1}{\varepsilon} = \frac{8\gamma}{\varepsilon^1} \]

\[ + \left[ \varepsilon_0 \frac{1}{1} \left( \frac{\varepsilon^1}{\varepsilon} \right) + \frac{\varepsilon^1}{\varepsilon} \right]_{12} \sec \frac{\varepsilon^1}{\varepsilon} = \frac{8\gamma}{\varepsilon^1} \]

Where

\[ \varepsilon^1 \] is the dielectric constant of the medium.

\[ \varepsilon \] is the index of refraction.

\[ \gamma \] is the refractive index of the medium.

Combining the results from the preceding developments, we may now write:

\[ \left( e_{10} \right) \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) + \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) + \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) = \frac{8\gamma}{\varepsilon^1} \]

Finally, in view of equations (59) and (62)

\[ \left( e_{10} \right) \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) + \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) + \frac{\varepsilon^1}{\varepsilon^1} \left( \frac{\varepsilon^1}{\varepsilon^1} \right) = \frac{8\gamma}{\varepsilon^1} \]

which added to the atmospheric component gives the total value of the integral.
Table 1A.

<table>
<thead>
<tr>
<th>Range Correction (mm)</th>
<th>Range (km)</th>
<th>0 - 10 km</th>
<th>11 - 20 km</th>
<th>21 - 30 km</th>
<th>31 - 40 km</th>
<th>41 - 50 km</th>
<th>51 - 60 km</th>
<th>61 - 70 km</th>
<th>71 - 80 km</th>
<th>81 - 90 km</th>
<th>91 - 100 km</th>
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<td></td>
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<td>0 - 10 km</td>
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<td>61 - 70 km</td>
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<td>公式 (1)</td>
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Table 1B.

<table>
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<th>Range Correction (mm)</th>
<th>Range (km)</th>
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<th>31 - 40 km</th>
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*Data converted to Table 1A.

Table 2A.

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<th>21 - 30 km</th>
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Table 2B.

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</table>

*Data converted to Table 2A.
in length between the measured arc and the corresponding chord. Thus the effective ray path is given due to the refraction. The difference

\[ \phi = s - s' \]

is expressed in millimeters, and \( T \) is the absolute temperature in degrees Fahrenheit.

where \( p \) is the total pressure and \( e \) the partial pressure of water vapor, both

\[ (1 - u)(1 - 0.07762) + (1 - 0.07762)/2 \]

into metric units:

formula adopted by the International Association of Geodesy [1963], converted

for the computation of these latter we shall have only the essential and frame

the case the group index is equal to the refraction index. Of the various forms

the non-directional, the group velocity will not propagate with a change of shape, in

independent. The atmosphere is considered

\[ f(x) = \int_a^b g(x) \, dx \]

due to the effect of the group velocity of waves.

where water depth \( \lambda \) is expressed in millimeters. Hence replacing

where water depth \( \lambda \) is expressed in millimeters. Hence replacing

\[ (1 - u) \left( \frac{\sqrt{n} - 1 - \sqrt{n}}{\sqrt{n} + 1 + \sqrt{n}} \right) = 1 - \lambda \]

repeatability of standard dev

of a Rayleigh distribution depends on the wave velocity. The

the group index must be used in electromagnetic distance measurement in

\[ \frac{\lambda}{n} \cdot \lambda - u \]

for the group index of refraction being obtained from refraction index \( n \). The

differentiation equation (39), which gives the following form for the group

velocity. It can be determined with sufficient accuracy by

by the propagation of the wave. The

formula

formula

Group Index of Refraction

Introduction to Pseudo-Computation of Range Correction

THEORY OF ATMOSPHERIC REFRACTION
must further be subtracted from the measured distance. The total range correction then becomes

\[ \Delta s = \Delta S + \delta_g \]  

(56)

where \( \delta_g \) is a geometric correction.

Referring to Figure 4, the geometric correction is approximately given by the integral

\[ \delta_g = \int_S \left[ 1 - \cos(\Delta z) \right] dS = \frac{1}{2} \int_{r_1}^{r'} (\Delta z)^2 \sec z \, dr \]

where \( \Delta z \) is the astronomical refraction at points along the ray path. Now from (30)

\[ \Delta z = \tan z (n - 1) - \frac{R}{rg} \tan z \sec^2 z (n - 1) T \]  

(57)

\[ \frac{1}{2} (\Delta z)^2 \sec z = \frac{1}{2} \tan^2 z \sec z (n - 1)^2 - \frac{R}{rg} \tan z \sec^3 z (n - 1)^2 T \]

and from (3) and (3*)

\[ \tan^2 z \sec z = \tan^2 z_1 \sec z_1 - \frac{3}{r_1} \sec^5 z_1 (r - r_1) \]

\[ \tan^2 z \sec^3 z = \sec^3 z_1 \]

approximately, which gives

\[ \delta_g = \frac{1}{2} \tan^2 z_1 \sec z_1 \int_{r_1}^{r'} (n - 1)^2 \, dr - \frac{3}{2r_1} \sec^5 z_1 \int_{r_1}^{r'} (n - 1)^2 (r - r_1) \, dr - \frac{R}{r_1 g} \sec^5 z_1 \int_{r_1}^{r'} (n - 1)^2 \, T \, dr \]  

(58)

The values of the first two integrals in equation (58) have been determined previously in equations (22), (23) and (52).

Equation (22) also gives the stratospheric component of the third integral in (58):

\[ \int_{r_1}^{r} (n - 1)^2 T \, dr = T^4 \int_{r_1}^{r'} (n - 1)^2 \, dr = \frac{R}{2g} (n_0 - 1)^2 T^2 \]

In the troposphere, we have from (17)

\[ \int (n - 1)^2 T \, dr = \frac{(n_1 - 1)^2 T_1}{\beta} \int \left( \frac{T}{T_1} \right)^{2m' + 1} \, dT = \]

\[ = \frac{(n_1 - 1)^2 T_1}{\beta (2m' + 2)} \left( \frac{T}{T_1} \right)^{1m' + 2} + C = \frac{R}{2g} (n - 1)^2 T^2 + C \]

and

\[ \int_{r_1}^{r_0} (n - 1)^2 T \, dr = \frac{R}{2g} \left[ (n_0 - 1)^2 T_1^2 - (n - 1)^2 T^2 \right] \]

The total value of the third integral is consequently
(69) $$\int_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

$$+ e \int_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

have the (69) we can derive a function for the correction of various humidity

(69) $$\int_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

for the purpose of correction.

To determine the contribution of humidity to effectively integrated (69),

(69) provides a convenient means for the correction of various humidity

where $$p$$ is a numerical coefficient determined from local observations. Equation

(19)

$$e \int (1 - \frac{1}{9}) = \int$$

and

The amount and distribution of water vapor in the atmosphere varies

Corrections for Vapor Pressure.

By modifying appropriate terms in equation (39):

(39)

$$\int_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

where the printed quantities, as before, refer to values corrected for ground

(69)

$$_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

:$$

\int_{\frac{9}{10}}^{1} \frac{8/d}{R} = 1 \int p \left( \frac{1}{9} \right) \frac{1}{z} \int$$

THEORY OF ATMOSPHERIC REFRACTION

J. STASMONEN
Formulas and Tables for the Computation of Range Correction


The atmospheric correction for troposphere and stratosphere in electromagnetic ranging of satellites is given by the standard formula (a) for laser ranging:

$$\Delta s_0 = 0.002357 \sec z \left( p + 0.06 e - B \tan^2 z \right) + \delta_L$$  \hspace{1cm} (56a)

or (b) for radio ranging:

$$\Delta s_0 = 0.002277 \sec z \left( p + \left( \frac{1255}{T} + 0.05 \right) e - B \tan^2 z \right) + \delta_R$$  \hspace{1cm} (56b)

where $\Delta s_0$ is the range correction in metres, $z$ is the apparent (radio) zenith distance of the satellite, $p$ is the total barometric pressure in millibars, $e$ is the partial pressure of water vapour in millibars, $T$ is the absolute temperature in degrees Kelvin, and $B$ and $\delta$ are correction quantities obtained from Tables X and XI, respectively. In radio ranging, apparent zenith distance $z$ can be determined from true zenith distance $Z$ of the satellite by the formula $z = Z - \Delta z$, where

$$\Delta z'' = \frac{16^{\circ}\cdot0 \tan Z}{T} \left( p + \frac{4800 e}{T} \right) - 0^{\circ}07 \left( \tan^2 Z + \tan Z \right) \left( \frac{p}{1000} \right)$$  \hspace{1cm} (57a)

is the angle of refraction.

2. Correction for the Effective Wavelength.

Formula (56a) employs a standard wavelength of 0.6943 microns for a ruby laser. For other laser systems, the numerical coefficient of the first term in the formula is obtained from the expression

$$0.39406 \left( \frac{173.3 + 1/\lambda^2}{173.3 - 1/\lambda^2} \right)$$

where $\lambda$ is the effective wavelength of the system expressed in microns.

3. Correction for Local Latitude and Station Height.

The numerical coefficient of the first term in formulas (56a) and (56b) is to some extent dependent on local latitude and station height. A locally corrected value may be obtained by applying a correction factor

$$1 + 0.0026 \cos 2 \psi + 0.00028 \text{H}$$
to obtain the necessary formulae. Let us denote the astronomical and theoretical values of the geodetic coordinates of the station A, B, and C, and L, respectively.

The problem of transmitting astronomical light—deflections along the horizontal plane—may be considered to consist of the following steps:

**1. Transmission of Optical Plumb Line Deflections**

**2. Accuracy of the Determination of Range Correction**