

## Maximum spectral luminous efficacy of white light

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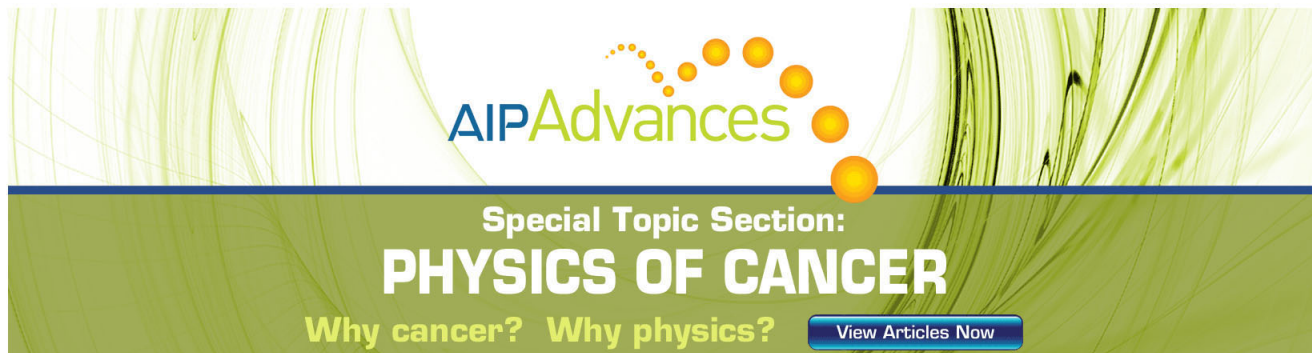
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## Maximum spectral luminous efficacy of white light

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As lighting efficiency improves, it is useful to understand the theoretical limits to luminous efficacy for light that we perceive as white. Independent of the efficiency with which photons are generated, there exists a spectrally imposed limit to the luminous efficacy of any source of photons. We find that, depending on the acceptable bandpass and—to a lesser extent—the color temperature of the light, the ideal white light source achieves a spectral luminous efficacy of 250–370 lm/W. This is consistent with previous calculations, but here we explore the maximum luminous efficacy as a function of photopic sensitivity threshold, color temperature, and color rendering index; deriving peak performance as a function of all three parameters. We also present example experimental spectra from a variety of light sources, quantifying the intrinsic efficacy of their spectral distributions. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4721897>]

### I. INTRODUCTION

Lighting technology has evolved rapidly in the past decade, enabling replacement of inefficient incandescent light bulbs with compact fluorescent lights (CFLs) and light-emitting diodes (LEDs). The figure of merit for the energy efficiency of lighting is *luminous efficacy*, measured in lumens per watt. Inefficiency can be separated into two main forms: inefficient production of photons from the input power source and distribution of photons outside of the visible spectrum. For example, incandescent lights are marvelously efficient at generating photons from the input electrical source, but produce the vast majority of these photons at near-infrared wavelengths where the human eye has no sensitivity. We can break the total luminous efficacy into spectral and electrical components, so that  $\eta_L = \eta_S \eta_E$ . The spectral part,  $\eta_S$  carries units of lm/W, while the electrical part,  $\eta_E$  is unitless, representing the ratio of luminous power output to electrical power input. This paper focuses on the spectral distribution aspect of lighting efficiency, and will not address the efficiency with which photons are produced.

A monochromatic source at 555 nm—the peak of the photopic sensitivity curve<sup>1</sup>—will produce 683 lm/W of light output.<sup>2</sup> For other monochromatic wavelengths, the luminous efficacy is reduced by a factor according to the photopic function. For instance, at the 633 nm wavelength of helium-neon lasers, the sensitivity of the eye is only 23.5% compared to its peak value (leading to 160 lm/W), whereas the human eye has 88.5% sensitivity at the 532 nm wavelength of frequency-doubled Nd:YAG lasers, corresponding to an efficacy of 604 lm/W.

But monochromatic illumination—as efficient as it might be—is often deemed to be unacceptable, providing no color differentiation. Even a composite of several line sources that may appear white-like can inadequately render some colored items, as can often be the case under the line-dominated spectrum of fluorescent lights.

Sporadic instances in the literature<sup>3,4</sup> place the maximum luminous efficacy for white light in the range 250–300 lm/W without reference to supporting material. Some papers (e.g., Ref. 5) display the appropriate integral along with a numerical result, but without detailing the integration limits or the specifics of the integrand. Uchida and Taguchi<sup>6</sup> estimate the theoretical luminous efficacy of a white LED with good color rendering to be  $\sim 300$  lm/W. Coltrin *et al.*<sup>7</sup> found that it is theoretically possible to synthesize a light achieving 408 lm/W with a color rendering index (CRI) in excess of 90 using four discrete wavelengths. But in practice the color rendering becomes unacceptable when the fluxes of the individual line sources are allowed to vary from the design by a standard deviation of 10%, facing the additional hurdle that the requisite narrow-line sources are not easily obtained. Yun-Li *et al.*<sup>8</sup> explored combinations of LED lights to synthesize white light, finding that luminous efficacies in excess of 300 lm/W and suitable color rendering could be achieved with three LEDs. None of these previous estimates are in error, yet they do not represent a comprehensive study of the limits to spectral luminous efficacy as a function of the quality characteristics of the light.

In this paper, we examine the theoretical limits to (spectral) luminous efficacy for lights that we would perceive as white, to varying degrees. We find that the maximum efficacy is in the range 250–370 lm/W, and that the color temperature of the light has only a modest impact within the range of values typically encountered. We explore lighting performance as a function of both the bandpass associated with a minimum threshold and color temperature. We also evaluate the maximum luminous efficacy as a function of color rendering index. Finally, we look at the spectral luminous efficacy of a variety of modern light sources. For what follows, references to luminous efficacy are to be interpreted as spectral luminous efficacy unless explicitly noted otherwise.

### II. METHOD OF CALCULATION

To assess the theoretical limits of spectral luminous efficacy, we integrate the normalized spectral density function,

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$B_\lambda$ , in fractional luminous power per meter (of wavelength), with the photopic sensitivity curve<sup>1</sup> of the eye,  $\bar{y}(\lambda)$ , which lies between 0.0 and 1.0, peaking at  $\lambda = 555$  nm:

$$\eta_S = 683 \int_0^\infty \bar{y}(\lambda) B_\lambda d\lambda. \quad (1)$$

The factor of 683 in front scales the result in accordance with the definition for the lumen.

The Sun is our standard “white” light source, and is well approximated by a Planck blackbody at the Sun’s surface temperature of 5800 K. The Planck function—normalized so that the integral over all wavelengths is unity—is

$$B_\lambda = 15 \left( \frac{hc}{\pi kT} \right)^4 \lambda^{-5} \left[ e^{hc/\lambda kT} - 1 \right]^{-1} \text{ m}^{-1}, \quad (2)$$

where  $\lambda$  is the wavelength,  $h$  is Planck’s constant,  $c$  is the speed of light,  $k$  is Boltzman’s constant, and  $T$  is the blackbody temperature, in Kelvin. A blackbody at the temperature of the Sun results in a luminous efficacy of  $\eta_S = 93$  lm/W. Only 37% of its light falls within the visible band from 400 nm to 700 nm. By comparison, a light bulb at 2800 K has a luminous efficacy—determined in the manner above—of 15 lm/W, with 6% of its light in the 400–700 nm band. Fig. 1 shows the luminous efficacy as a function of blackbody temperature, peaking at 6640 K and 96.1 lm/W. As a caveat that does not impact the conclusions of this paper, we note that actual tungsten filaments achieve 15 lm/W at lower temperatures around 2500 K—being imperfect blackbodies that selectively emit light in the visible compared to the infrared part of the spectrum.

To evaluate the color rendering performance of each source, we compute the correlated color temperature (CCT), offset from “white,” and CRI for the spectra considered herein—following the procedure outlined by the International Commission on Illumination (CIE).<sup>9</sup> In brief, the procedure involves establishing the chromatic coordinates of the source within the CIE 1960  $uvY$  color space, and relating this to the chromatic locus of points corresponding to blackbody radiation profiles. The CCT is the closest black-

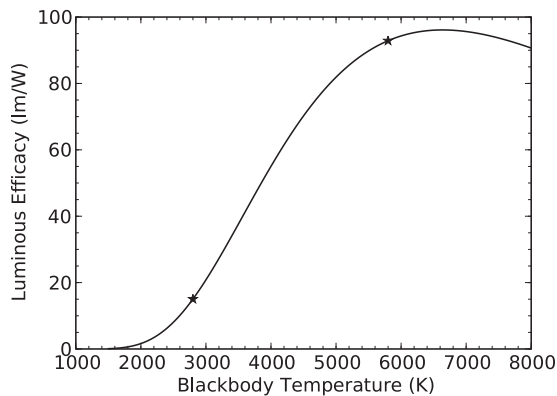


FIG. 1. Luminous efficacy of pure blackbody radiation as a function of temperature. The effective temperatures of a typical incandescent filament and the solar surface are marked by stars, corresponding to 15 and 93 lm/W, respectively.

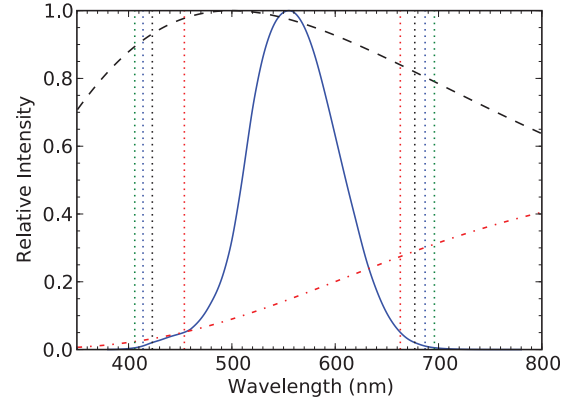


FIG. 2. The photopic sensitivity curve is shown as the solid curve. Blackbodies corresponding to 5800 K (dashed) and 2800 K (dashed-dotted), normalized to the same total radiant flux, are shown for reference. Dotted vertical lines represent different bounds within which we consider artificial ideal white light sources, corresponding to photopic curve intercepts at 0.5%, 1%, 2%, and 5% sensitivity levels.

body temperature in color space, and the Pythagorean distance between the points indicates how “white” the source looks ( $>5.4 \times 10^{-3}$  is considered to be too far for the CRI computation to be reliable). Establishing the CRI is substantially more involved, and is based on comparing a standard set of 8 Munsell colors under illumination by both the spectrum under study and by a blackbody at the CCT—ultimately forming the offset metric in the CIE 1964  $UVW$  color space. The CRI ranges from 0 to 100, with values above 90 typically considered to be adequate for general lighting. The CRI as calculated in this way is a flawed construct, but nonetheless is in common practice and is used here for comparative purposes.

If we construct a *truncated* 5800 K blackbody so that it emits light only in the range between cutoff wavelengths  $\lambda_{\text{blue}}$  and  $\lambda_{\text{red}}$ , the luminous efficacy is now given by

$$\eta_S = 683 \frac{\int_{\lambda_{\text{blue}}}^{\lambda_{\text{red}}} \bar{y}(\lambda) B_\lambda d\lambda}{\int_{\lambda_{\text{blue}}}^{\lambda_{\text{red}}} B_\lambda d\lambda}, \quad (3)$$

TABLE I. Properties of truncated continuum sources.

Source	Photopic cutoff (%)	CCT (K)	$\eta_S$ (lm/W)	Planckian offset	CRI
5800 K blackbody	0.5	5784	260	$0.64 \times 10^{-3}$	99.4
	1	5756	274	$1.3 \times 10^{-3}$	98.8
	2	5653	292	$3.4 \times 10^{-3}$	96.5
	5	4646	348	$25 \times 10^{-3}$	68.6
	2800 K blackbody	0.5	2811	256	$0.14 \times 10^{-3}$
2800 K blackbody	1	2821	276	$0.46 \times 10^{-3}$	98.7
	2	2839	299	$1.3 \times 10^{-3}$	97.2
	5	2831	343	$10 \times 10^{-3}$	82.7
uniform $B_\lambda$	0.5	5440	253	$3.7 \times 10^{-3}$	96.4
	1	5415	268	$2.9 \times 10^{-3}$	97.1
	2	5324	287	$0.71 \times 10^{-3}$	98.4
	5	4418	344	$21 \times 10^{-3}$	71.5

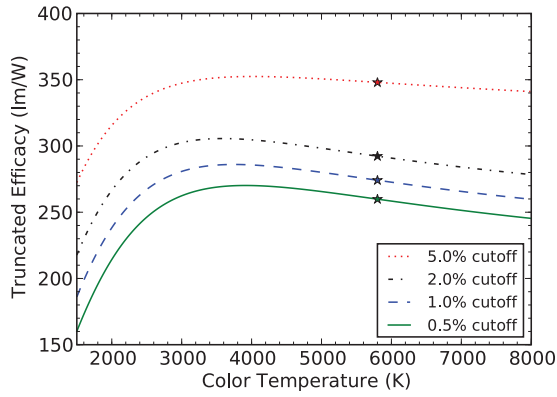


FIG. 3. Luminous efficacy of ideal, truncated sources for four different wavelength ranges as a function of blackbody temperature. The four wavelength ranges corresponding to 0.5%, 1%, 2%, and 5% sensitivity thresholds of the photopic response curve, whose wavelengths are indicated by vertical dotted lines in Fig. 2. Stars mark the efficacies at 5800 K.

and evaluates to 251 lm/W for a 5800 K blackbody truncated to emit light only between  $\lambda_{\text{blue}} = 400 \text{ nm}$  and  $\lambda_{\text{red}} = 700 \text{ nm}$ . This describes one manifestation of the ideal white light, which might in principle be synthesized out of narrow emission sources, such as LEDs, over a range of wavelengths.

More sensible would be to base the  $\lambda_{\text{blue}}$  and  $\lambda_{\text{red}}$  limits on human photopic sensitivity levels, rather than on the arbitrary—although convenient—400 nm and 700 nm values. For the analysis in this paper, we principally concentrate on

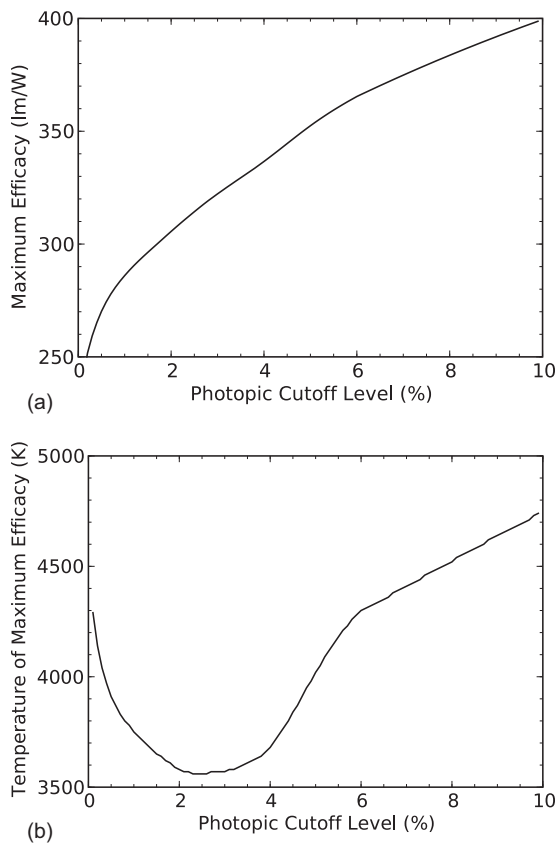


FIG. 4. Maximum luminous efficacy achievable as a function of photopic sensitivity threshold (a) and the corresponding color temperature (b).

the part of the photopic response that lies above 0.5%, 1%, 2%, and 5% of peak sensitivity. These threshold levels correspond to  $\lambda_{\text{blue}}$  values of 405.7, 413.2, 422.3, and 453.2 nm, respectively, and to  $\lambda_{\text{red}}$  values of 697.0, 687.4, 677.4, and 663.2 nm, respectively. Fig. 2 illustrates these ranges in relation to the photopic sensitivity curve, together with blackbody curves at 2800 K and 5800 K for reference.

### III. RESULTS

Using the four principal wavelength ranges discussed above, the ideal luminous efficacy for light following a 5800 K spectrum computes to  $\eta_S = 260, 274, 292,$  and  $348 \text{ lm/W}$ , corresponding to 0.5%, 1%, 2%, and 5% sensitivity thresholds, respectively. Table I lists additional properties of these truncated spectra, together with two other cases. The results are rather similar for a truncated 2800 K source. We see in Table I some justification for stopping at a 5% photopic cutoff, as the CRI for these cases abruptly enters a region considered to be unacceptable in the lighting industry. Perhaps more importantly, the  $u-v$  chromaticity offset for these cases exceeds the threshold of  $5.4 \times 10^{-3}$ , and therefore would no longer be considered “white.” It should be noted that the CRI is not customarily compared among sources with differing color temperatures.

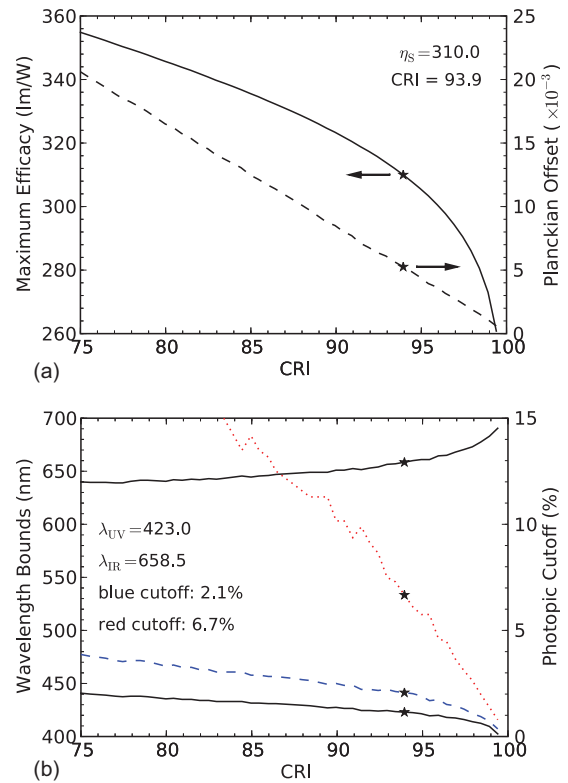


FIG. 5. Conditions for maximum luminous efficacy for a truncated blackbody at 5800 K, allowing asymmetric red and blue cutoff wavelengths. Panel (a) shows the maximum luminous efficacy (solid) and associated Planckian offset (dashed) as a function of CRI achieved. The star marks the point at which the Planckian offset reaches  $5.4 \times 10^{-3}$  and is no longer considered to be “white.” Panel (b) shows the wavelength cutoffs (solid curves) that maximize  $\eta_S$ , and their corresponding photopic cutoff sensitivities (dashed for blue, dotted for red). Numbers correspond to the stars, which themselves indicate the point at which the Planckian offset reaches  $5.4 \times 10^{-3}$ .



Another case presented in Table I is that of a truncated spectrally uniform light source, with constant  $B_\lambda$  across the range from  $\lambda_{\text{blue}}$  to  $\lambda_{\text{red}}$ , and no light outside of this range. For the four principal sensitivities examined here, the uniform case delivers 253, 268, 287, and 344 lm/W. These numbers are very similar to the blackbody cases considered above. It is interesting that the Planckian offset (departure from “white”) and the CRI initially improve as the photopic cutoff increases. The uniform spectrum is red- and blue-heavy compared to a 5800 K blackbody, so that increasing truncation initially compensates this overabundance of light at the extremes. All the same, by the time one reaches a 5% photopic cutoff, the light is no longer acceptable, either in terms of Planckian offset or CRI.

We have become accustomed to incandescent lighting, and have developed some affinity for the “warm” color temperatures they emit. A daylight spectrum presented at night may be perceived as jarring and harsh. Investigating the maximum luminous efficacy for lights of different color temperatures reveals a surprise—perhaps partly exposed by the similarity of numbers in the two blackbody and uniform spectral density cases above. The maximum luminous efficacy in a truncated spectral source is largely independent of color temperature, as seen in Fig. 3.

Between color temperatures of 2500–8000 K, the maximum luminous efficacy for a particular spectral cutoff varies by less than 10%. It is interesting to ascertain the maximum efficacy achievable as a function of sensitivity threshold, as well as the temperature at which maximum efficacy is

realized. The result is presented in Fig. 4. The reversal in peak color temperature as the photopic cutoff increases beyond about 3% can be seen as a result of the flattening curves in Fig. 3. The maximum of each curve first shifts toward lower color temperatures as the photopic cutoff increases, but then the curves begin to flatten at high color temperatures, pushing the maximum to higher temperatures. The break seen at 6% in both panels of Fig. 4 relates to the point at which the blue tail of the photopic curve abruptly transitions to a higher slope.

Relaxing the constraint that  $\lambda_{\text{blue}}$  and  $\lambda_{\text{red}}$  be determined by symmetric thresholds on the photopic sensitivity curves, we can ask what combination of  $\lambda_{\text{blue}}$  and  $\lambda_{\text{red}}$  delivers the maximum luminous efficacy as a function of achieved CRI. For instance, we can demand a CRI of 95 and seek the highest  $\eta_S$  that can result from a blackbody function truncated by arbitrary  $\lambda_{\text{blue}}$  and  $\lambda_{\text{red}}$ . Fig. 5 shows the result for a truncated 5800 K blackbody. The Planckian offset exceeds  $5.4 \times 10^{-3}$  for CRI values below  $\sim 94$ , at which point the spectral luminous efficacy is  $\eta_S \approx 310$  lm/W. Exploring what this means in terms of wavelength cutoff, we find that the maximum luminous efficacy favors sacrificing red light sooner than giving up blue (right panel of Fig. 5). In this case, the blue cutoff is at 423 nm, corresponding to 2.1% photopic sensitivity. Meanwhile, the red cutoff is at 658.5 nm, or a photopic sensitivity of 6.7%. Similar trends are found for a truncated 2800 K spectrum, as seen in Fig. 6—except that the light

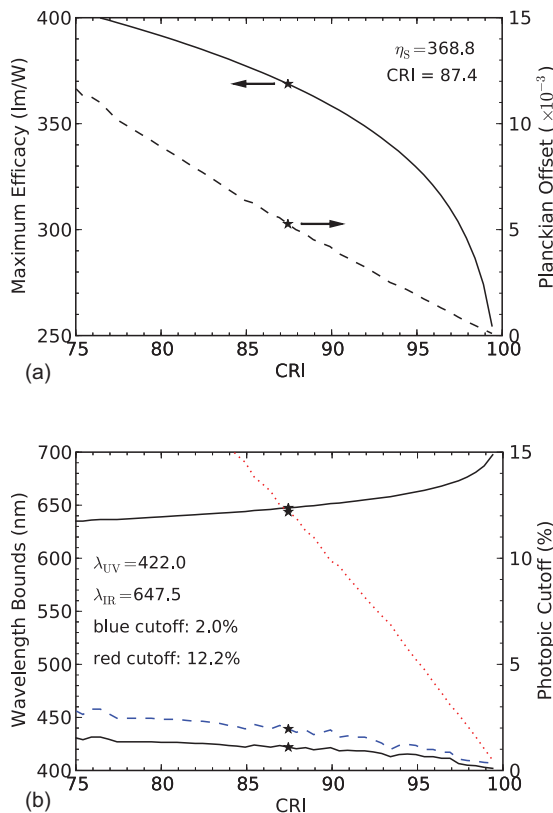


FIG. 6. Conditions for maximum luminous efficacy for a truncated blackbody at 2800 K, allowing asymmetric red and blue cutoff wavelengths. Conventions follow those from Fig. 5.

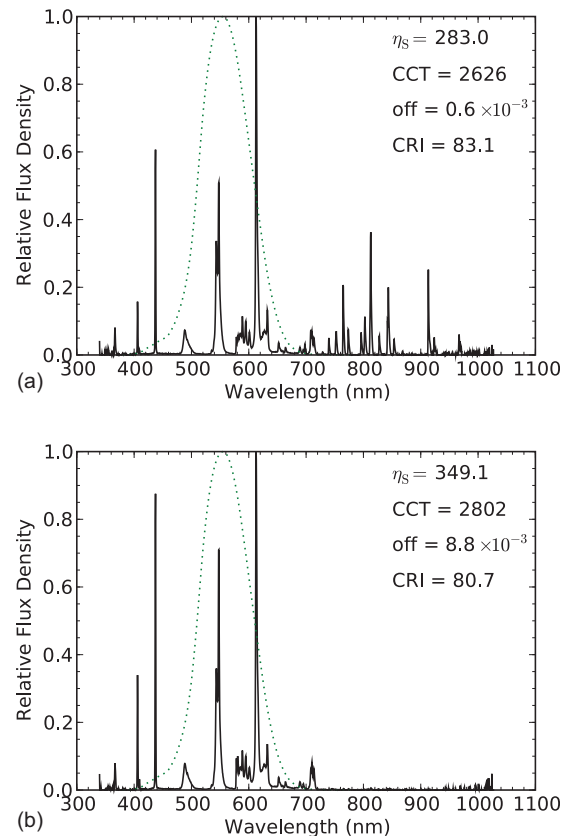


FIG. 7. 16 W CFL (“60 W” replacement) at turn-on (a) and after warm-up (b). The spectral luminous efficacy, correlated color temperature, Planckian offset, and color rendering index are displayed for each spectrum. The photopic sensitivity curve is also displayed for reference.

remains “white” at a lower CRI, pushing the luminous efficacy up to  $\sim 370$  lm/W, sacrificing even more red light in the red-tilted spectrum.

Exploring one final scenario, one may be tempted to match a light source to the photopic sensitivity curve, because not much light would be “wasted” at wavelengths to which the human eye is not sensitive. Such a source can be approximated by a Gaussian curve centered at 560 nm and a full width at half-maximum of 100 nm. This light would deliver a luminous efficacy of 488 lm/W, but would look distinctly green, being similar in light distribution to that of green LED, albeit with a broader spectrum. In particular, the Planckian offset is 0.054, and the CRI computes to 24: a grossly inadequate source of white light.

**IV. CHARACTERIZATION OF REAL SOURCES**

We can evaluate today’s sources of alternative lighting by acquiring spectral distributions and computing the corresponding spectral luminous efficacy. This technique does not address the efficiency with which electrical power input is converted to luminous energy,  $\eta_E$ , but simply evaluates the efficacy stemming from the distribution,  $\eta_S$ .

Spectra were obtained using the USB2000 spectrometer from Ocean Optics, capturing 12-bit raw data at a spectral resolution between 400 and 500 and sampling ranging from 0.38 nm at the UV end to 0.28 nm at the IR end. Spectra are

dark-subtracted and calibrated against a solar spectrum acquired by the same setup that itself is fitted to a 5800 K Planck function, avoiding absorption features from the solar and terrestrial atmospheres in the fitting procedure.

Luminous efficacies are evaluated in the wavelength range from 400 to 900 nm, which does not catch 100% of the light in all cases (and would be a poor choice for blackbody sources), but captures the vast majority of the light output for the sources presented here.

Fig. 7 shows the spectrum of a 16 W compact fluorescent light rated at 900 lm as it appears seconds after turn-on from an ambient temperature state and after settling in a warm equilibrium. The luminous efficacy of the spectral distribution evolves from 283 lm/W to 349 lm/W as many of the infrared lines disappear and the green line achieves greater relative dominance. In the process, the Planckian offset exceeds the acceptable limit, and the CRI is well short of the target of 90. At  $\eta_L = 56$  lm/W and  $\eta_S = 349$  lm/W, the inferred electricity-to-light ratio is  $\eta_E \sim 16\%$ .

Fig. 8 illustrates the spectral distributions of two lights using similar total electrical power and based on the same blue LED, but using different phosphors at differing levels of absorption to achieve advertised color temperatures of 3000 K and 6500 K. The spectral distributions achieve luminous efficacies of 341 lm/W and R287 lm/W, respectively. While both lights closely bracket the acceptable Planckian offset, neither have a high CRI—especially the 3000 K light. The 3000 K light is rated at 86 lm, implying an electrical-to-light efficiency of  $\eta_E \sim 17\%$ .

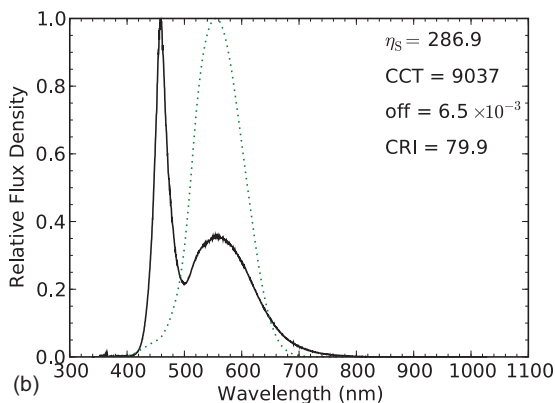
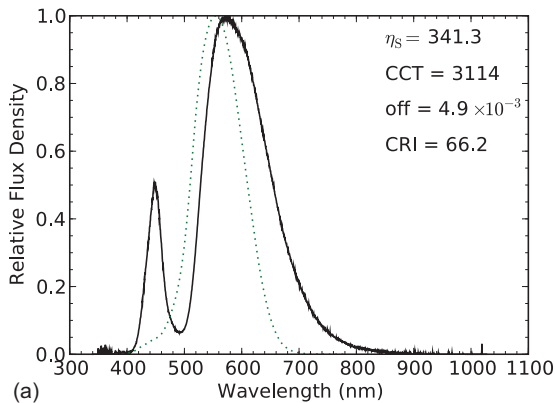


FIG. 8. 1.5 W LED lights labeled as having color temperatures of 3000 K (a) and 6500 K (b). The measured color temperature of the light in (b) is clearly a poor match to that indicated on the packaging. Conventions and labels follow that of Fig. 7.

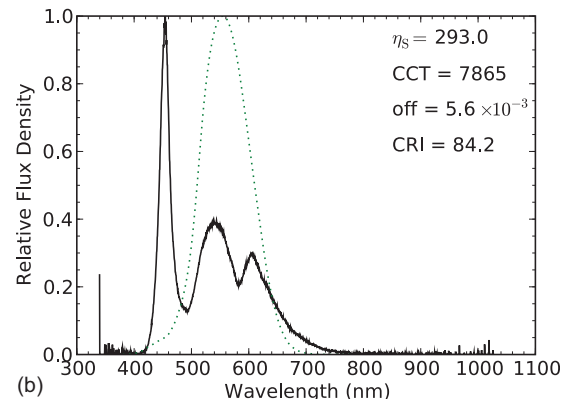
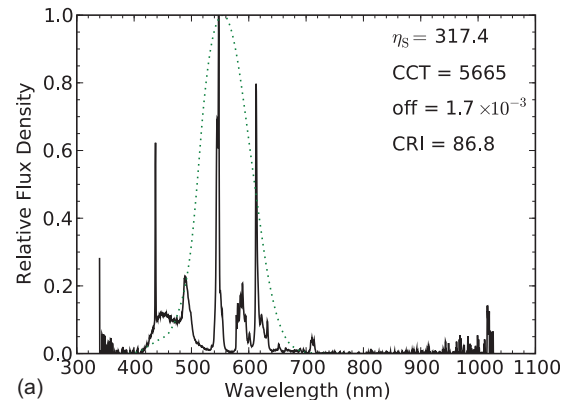


FIG. 9. Backlighting for laptop computer liquid crystal displays, showing fluorescent tube illumination (a) and LED illumination (b). Conventions and labels follow that of Fig. 7.

Fig. 9 presents spectra from the white background of two different laptop computer displays backlit by fluorescent tubes and LEDs, achieving spectral luminous efficacies of 317 lm/W and 293 lm/W, respectively. The spectra differ qualitatively from their lighting counterparts, chiefly in their use of phosphors. Spectra were also acquired for a LED-illuminated television, scoring 283 lm/W and appearing qualitatively identical to the spectrum of the LED computer display. Each of the displayed spectra has reasonably small Planckian offsets, and the CRI values are substantially higher than for the lighting counterparts in Figs. 7 and 8.

Each of these sources—by confining emission to the visible parts of the spectrum—are capable of far better spectral luminous efficacies than are incandescent sources. Each of the spectra presented here land within the range 250–350 lm/W, despite vastly different spectral distributions. For clarity, these lights do not achieve total luminous efficacies above 100 lm/W due to inefficiencies in the *generation* of photons. Given that all of the measured spectra in Figs. 7–9 yield spectral efficacies in the 280–350 lm/W range, we infer that generation efficiencies above about  $\eta_E \sim 0.3$  in lights sharing similar spectra will naturally deliver overall luminous efficacies above 100 lm/W.

## V. CONCLUSIONS

Synthesizing a white light source that emits no light outside the visible band can achieve luminous efficacies in a range between 250 and 370 lm/W depending on spectral extent and corresponding “whiteness.” One approaches the upper end of this range when truncating the spectrum at the 5% photopic sensitivity limits of the eye, generating light only between 453 and 663 nm. But at this cutoff, the light is already inadequate in terms of color rendering index and Planckian offset. By accepting asymmetric cutoffs, we are able to achieve adequate color properties at  $\eta_S \approx 310$  lm/W

for a color temperature of 5800 K, and  $\eta_S \approx 370$  lm/W at 2800 K. At the high-quality end, it is unlikely that any white-light application for humans would require illumination at the very low end of the photopic sensitivity curve, below 0.5% (corresponding to wavelength cutoffs at 406 and 697 nm), so that 250 lm/W can be taken as the lower bound to maximum practical luminous efficacy of a white source.

Unlike the pure blackbody, which achieves a maximum luminous efficacy around 6640 K, a truncated source performs best between 3500 and 4100 K. Even so, the spectral efficacy is relatively constant (within 10%) across color temperatures ranging from 2500 to 8000 K for a given set of truncation bounds. Similarly, lower color temperatures produce higher spectral luminous efficacies under asymmetric cutoff wavelengths.

Leaving aside the efficiency with which photons are generated, any real white-like spectrum confining itself to visible wavelengths is likely to achieve a *spectral* luminous efficacy in the range 250–350 lm/W, as is demonstrated by a wide variety of sources above. This fact emphasizes that improvements in the efficacy of current lighting technology must primarily involve advances in photon generation efficiency rather than spectral conditioning.

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