Maximum Efficiency of White Light

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Light sources require power input, P_{in}, measured in Watts. The output is a stream of photons, each carrying a finite amount of energy, $E_\gamma = h\nu$, where $h = 6.626 \times 10^{-34}$ J·s, and ν is the frequency of the light, in Hz. The emission of N photons in time interval τ therefore constitutes an output power of $P_{\text{out}} = NE_\gamma/\tau$. One may be tempted to claim that the light's efficiency is then just $\varepsilon = P_{\text{out}}/P_{\text{in}}$. But this is only part of the story: not all the photons are equally visible to us—if at all.

The function of a light source is typically to assist human vision. An appropriate measure of light level as sensed by the human eye is the *lumen*, defined in relation to the photopic (bright) response of the eye. The photopic curve peaks at 555 nm, which is green. A monochromatic source at 555 nm will produce 683 lumens per Watt of light output. This measure, lm/W, is called *luminous efficacy*. For other monochromatic wavelengths, the lighting efficacy is reduced by a factor according to the sensitivity curve for the eye. For instance, at the wavelength of the helium-neon laser at 633 nm, the sensitivity of the eye is only 23.5% of what it is at the peak (leading to 160 lm/W), whereas the green neodymium:YAG lasers at 532 nm come in at 88.5% sensitivity, for 604 lm/W.

But monochromatic illumination—as efficient as it might be—is not very pleasant, yielding no color differentiation. Even a composite of several line sources that may appear white-like can render some colored items in unfortunate ways, as can often be the case under the line-dominated fluorescent light spectrum. Natural light tends to be distributed continuously over wavelengths, rather than clumped into discrete-wavelength sources.

How do we compute the luminous efficacy of continuous-wavelength sources? We integrate the spectral density function, B_λ, in fractional contribution per meter (of wavelength), for instance, times the photopic sensitivity curve of the eye, $\bar{y}(\lambda)$ [1], which lies between 0.0 and 1.0, peaking at $\lambda = 555$ nm:

$$\epsilon = 683 \int_0^\infty \bar{y}(\lambda) B_\lambda d\lambda.$$

The factor of 683 out front scales the result in accordance with the definition for the lumen.

The sun is our standard "white" light source, and is well approximated by a Planck blackbody at the sun's surface temperature of 5800 K. The Planck function—normalized so that the integral over all wavelengths is unity—is:

$$B_\lambda = \left(\frac{hc}{\pi kT}\right)^4 \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \ \text{m}^{-1},$$

where h is Planck's constant (from above), $c = 2.998 \times 10^8$ m/s, $k = 1.38 \times 10^{-23}$ J/K, and T is the blackbody temperature, in Kelvin.
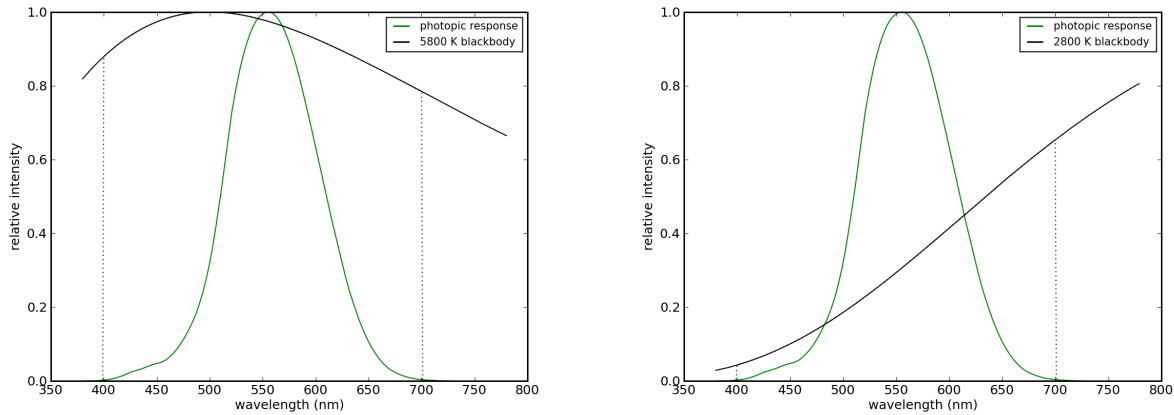
Figure 1: Spectral appearance of the solar blackbody (left) and an incandescent light blackbody (right). The photopic sensitivity curve is seen as the green curve in both. Both blackbody functions are scaled to peak at 1.0 in the figures. The truncated version is confined to exist only within the dashed vertical lines, since our eyes have essentially no photopic response outside this range.

A blackbody at the temperature of the sun results in a luminous efficacy of 93 lm/W. Only 37% of its light is within the visible band from 400 nm to 700 nm (Figure 1). If we were able to truncate a 5800 K blackbody so that it emits *no* light outside these limits, the luminous efficacy would become 251 lm/W. This describes the ideal white light, which might in principle be synthesized out of narrow emission sources over a range of wavelengths. By comparison, a light bulb at 2800 K has a luminous efficacy—determined in the manner above—of 15 lm/W, with 6% of the light in the 400–700 nm band (Figure 1).

Note that an incandescent light bulb has nearly 100% efficiency in terms of $P_{\mathrm{in}} = P_{\mathrm{out}}$, in that the input energy is almost perfectly converted into outgoing photon energy. It's just that most of these photons are in the near-infrared portion of the spectrum and do not aid our vision. In contrast, a modern LED lighting module will experience some inefficiency in that the LED power input is not all converted to photons (the devices warm up and emit at mid-infrared wavelengths). Also, conditioning AC electrical input into DC for the LEDs is not perfectly efficient. But even so, the fact that almost all of the LED light is visible is a huge improvement over incandescent bulbs, routinely achieving luminous efficacies of 60 lm/W or better.

This analysis points out that the most perfectly engineered light that we would perceive as "white" cannot achieve much more than about 250 lm/W.

References

[1] http://www.cvrl.org/database/text/lum/vljv.htm http://www.cvrl.org/database/text/lum/vljv.htm