Rockets, Orbits, and Universal Gravitation

Some Questions We'll Address Today

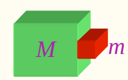
- What makes a rocket go?
- How can a rocket work in outer space?
- How do things get into orbit?
- What's special about geo-synchronous orbit?
- How does the force of gravity depend on mass and separation?

What does a rocket push against?

- Cars push on the road
- Boats push on the water
- Propellers push against air
- Jet engines push air through turbines, heat it, and push against the hot exhaust (air)
- What can you push against in space?

Momentum is conserved!

- Before



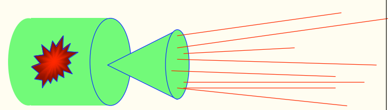
$v = 0$ so $p = 0$

- After

$p_{after} = Mv_1 + mv_2 = 0$ as well so
$v_1 = -(m/M)\, v_2$

A Rocket Engine: The Principle

- Burn Fuel to get hot gas
  - hot = thermally fast → more momentum
- Shoot the gas out the tail end
- Exploit momentum conservation to accelerate rocket



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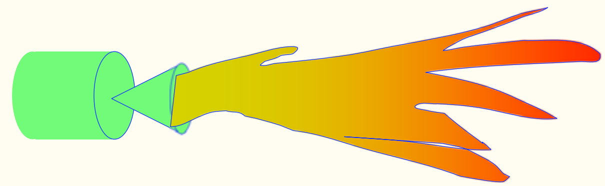


Rockets push against the inertia of the ejected gas!

- Imagine standing on a sled throwing bricks.
  - Conservation of momentum, baby!
- Each brick carries away momentum, adding to your own momentum
- Can eventually get going *faster* than you can throw bricks!
  - In this case, a stationary observer views your thrown bricks as also traveling forward a bit, but not as fast as you are

What counts?

- The “figure of merit” for propellant is the momentum it carries off, mv.
- It works best to get the propulsion moving as fast as possible before releasing it
- Converting fuel to a hot gas gives the atoms speeds of around 6000 km/h!
- Rockets often in stages: gets rid of “dead mass”
  - same momentum kick from propellant has greater impact on velocity of rocket if the rocket’s mass is reduced

Spray Paint Example

- Imagine you were stranded outside the space shuttle and needed to get back, and had only a can of spray paint. Are you better off throwing the can, or spraying out the contents? Why?
  - Note: Spray paint particles (and especially the gas propellant particles) leave the nozzle at 100-300 m/s (several hundred miles per hour)

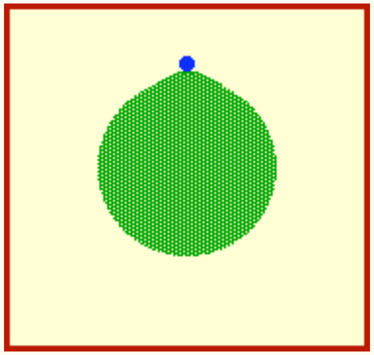
Going into orbit

- Recall we approximated gravity as giving a const. acceleration at the Earth's surface
  - It quickly reduces as we move away from the sphere of the earth
- Imagine launching a succession of rockets upwards, at increasing speeds
- The first few would fall back to Earth, but eventually one would escape the Earth's gravitational pull and would break free
  - Escape velocity from the surface is 11.2 km/s

Going into orbit, cont.

- Now launch sideways from a mountaintop
- If you achieve a speed v such that $v^2/r = g$, the projectile would orbit the Earth at the surface!
- How fast is this? $R_{\oplus} \sim 6400$ km = 6.4×10^6 m, so you'd need a speed of sqrt[$(6.4\times10^6$m)(10m/s^2)] = sqrt (6.4×10^7) m/s, so:
  - $v \approx 8000$ m/s = 8 km/s = 28,800 km/hr ~ 18,000 mph

4 km/s: Not Fast Enough....



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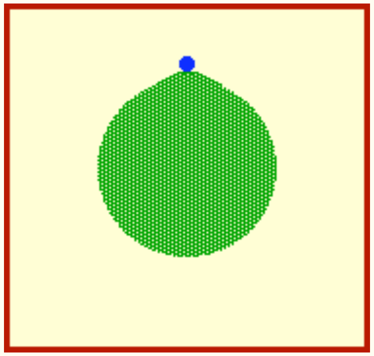
6 km/s: Almost Fast Enough....but not quite!



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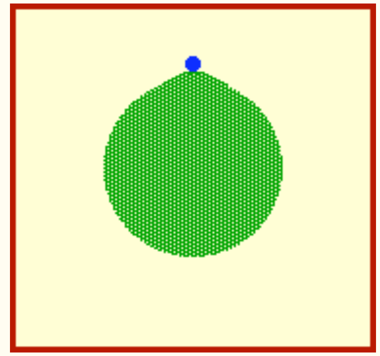
8 km/s: Not Too Fast, Nor Too Slow....Just Right



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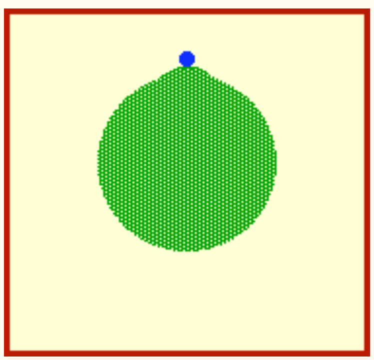
10 km/s: Faster Than Needed to Achieve Orbit



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Newton's Law of Universal Gravitation

The Gravitational Force between two masses is proportional to each of the masses, and inversely proportional to the square of their separation.

$$F = GM_1M_2/r^2$$

Newton's Law of Universal Gravitation

$a_1 = F/M_1 = GM_2/r^2 \rightarrow$ acceleration of mass #1 due to mass #2

(remember when we said grav. force was proportional to mass?)

$G = 6.674\times10^{-11}$ m^3/(kg·s^2)

Earth: $M = 5.976\times10^{24}$ kg; $r = 6{,}378{,}000$ m $\rightarrow a = 9.80$ m/s^2

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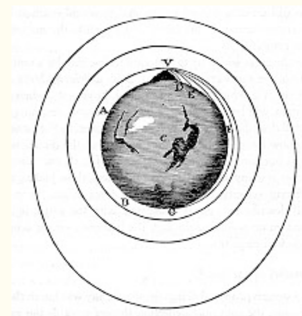
What up, G?

- *G* is a constant we have to shove into the relationship to match observation
  - Determines the strength of gravity, if you will
- *Best* measurement of *G* to date is 0.001% accurate
- Large spheres attract small masses inside canister, and deflection is **accurately** measured

Newton's classic picture of orbits



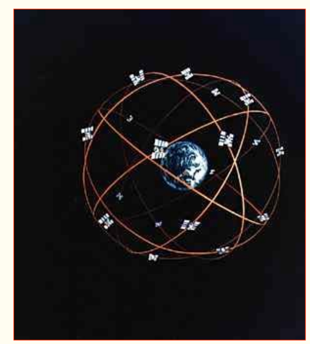
- Low-earth-orbit takes 88 minutes to come around full circle
- Geosynchronous satellites take 24 hours
- The moon takes a month
- Can figure out circular orbit velocity by setting $F_{gravity} = F_{centripetal}$, or:
  - $GMm/r^2 = mv^2/r$, reducing to $v^2 = GM/r$
  - M is mass of large body, r is the radius of the orbit

Space Shuttle Orbit

- Example of LEO, Low Earth Orbit ~200 km altitude above surface
- Period of ~90 minutes, v = 7,800 m/s
- Decays fairly rapidly due to drag from small residual gases in upper atmosphere
  - Not a good long-term parking option!

Other orbits

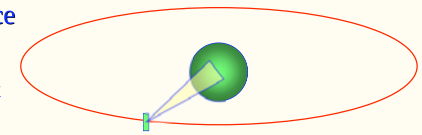
- MEO (Mid-Earth Orbits)
  - Communications satellites
  - GPS nodes
  - half-day orbit 20,000 km altitude, v = 3,900 m/s
- Elliptical & Polar orbits
  - Spy satellites
  - Scientific sun-synchronous satellites



GPS Constellation

Geo-synchronous Orbit

- Altitude chosen so that period of orbit = 24 hrs
  - Altitude = 36,000 km (~ 6 $R_\oplus$), v = 3,000 m/s
- Stays above the same spot on the Earth!
- Only equatorial orbits work
  - That's the direction of earth rotation
- Scarce resource
- Cluttered!
  - 2,200 in orbit



Rotating Space Stations Simulate Gravity

From 2001: A Space Odyssey rotates like bicycle tire

- Just like spinning drum in amusement park, create gravity in space via rotation
- Where is the "floor"?
- Where would you still feel weightless?
- Note the windows on the face of the wheel

Summary

- Rockets work through the conservation of momentum – "recoil" – the exhaust gas does not "push" on anything
- $F = GMm/r^2$ for the gravitational interaction
- Orbiting objects are often in uniform circular motion around the Earth
- Objects seem weightless in space because they are in free-fall *around* earth, along with their spaceship
- Can generate artificial gravity with rotation

Assignments

- HW for 2/17: 7.E.42, 7.P.9; 6.R.16, 6.R.19, 6.R.22, 6.R.23, 6.E.8, 6.E.12, 6.E.43, 6.P.6, 6.P.12, 8.R.29, 8.E.47, 8.P.9, plus additional questions accessed through website