Basic Circuit Analysis

- What we won’t do:
  - common electronics-class things: RLC, filters, detailed analysis
- What we will do:
  - set out basic relations
  - look at a few examples of fundamental importance (mostly resistive circuits)
  - look at diodes, voltage regulation, transistors
  - discuss impedances (cable, output, etc.)

The Basic Relations

- \( V \) is voltage (volts: V); \( I \) is current (amps: A); \( R \) is resistance (ohms: \( \Omega \)); \( C \) is capacitance (farads: F); \( L \) is inductance (henrys: H)
- Ohm’s Law: \( V = IR \)
- Power: \( P = IV \)
- Resistors and inductors in series add
- Capacitors in parallel add
- Resistors and inductors in parallel, and capacitors in series add according to:

\[
\frac{1}{X_{eq}} = \frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3} + \ldots
\]

Example: Voltage divider

- Voltage dividers are a classic way to set a voltage
- Works on the principle that all charge flowing through the first resistor goes through the second
  - So \( \Delta V = R \text{-value} \)
  - provided any load at output is negligible: otherwise some current goes there too
- So \( V_{out} = V \left( \frac{R_2}{R_1 + R_2} \right) \)
- \( R_1 \) is a variable resistor, or potentiometer, or “pot”
  - typically three terminals: \( R_{13} \) is fixed, tap slides along to vary \( R_{13} \) and \( R_{23} \), though \( R_{13} + R_{23} = R_{13} \) always
Real Batteries: Output Impedance

- A power supply (battery) is characterized by a voltage (V) and an output impedance (R)
  - sometimes called source impedance
- Hooking up to load: \( R_{\text{load}} \) we form a voltage divider, so that the voltage applied by the battery terminal is actually \( V_{\text{out}} = V(R_{\text{load}}/(R+R_{\text{load}})) \)
  - thus the smaller \( R \) is, the "stiffer" the power supply
- When \( V_{\text{out}} \) is a source impedance or output impedance
  - \( V_{\text{out}} \) may vary with load, though (not a real resistor)

Diode Example: 6A out of 1.5 V battery indicates 0.25 \( \Omega \) output impedance

Power Supplies and Regulation

- A power supply typically starts with a transformer
  - to knock down the 340 V peak-to-peak (120 V AC) to something reasonable/ manageable
- We will be using a center-tap transformer
  - \((A' - B') = (\text{winding ratio}) \times (A - B)\)
  - when \( A > B \), so is \( A' > B' \)
  - geometry of center tap (CT) guarantees it is midway between \( A' \) and \( B' \) (frequently tie this to ground so that \( A' = -B' \))
  - note that secondary side floats: no ground reference built-in

Diodes

- Diodes are essentially one-way current gates
- Symbolized by:
- Current vs. voltage graphs:
  - plain resistor
  - diode
  - idealized diode
  - \( 0.6 \text{ V} \)
  - \( I \) acts just like a wire (will support arbitrary current) provided that voltage is positive
  - \( I' \) the direction the arrow points in the diode symbol is the direction that current will flow

Diode Makeup

- Diodes are made of semiconductors (usually silicon)
- Essentially a stack of \( p \)-doped and \( n \)-doped silicon to form a \( p-n \) junction
  - doping means deliberate impurities that contribute extra electrons (\( n \)-doped) or "holes" for electrons (\( p \)-doped)
- Transistors are \( n-p-n \) or \( p-n-p \) arrangements of semiconductors
LEDs: Light-Emitting Diodes

- Main difference is material is more exotic than silicon used in ordinary diodes/transistors
- Typically 2-volt drop instead of 0.6 V drop
- When electron flows through LED, loses energy by emitting a photon of light rather than vibrating lattice (heat)
- LED efficiency is 30% (compare to incandescent bulb at 10%)
- Must supply current-limiting resistor in series:
  - Figure on 2 V drop across LED; aim for 1–10 mA of current

Getting DC back out of AC

- AC provides a means for us to distribute electrical power, but most devices actually want DC
  - Bulbs, toasters, heaters, fans don’t care: plug straight in
  - Sophisticated devices care because they have diodes and transistors that require a certain polarity
    - Rather than oscillating polarity derived from AC
    - This is why battery orientation matters in most electronics
- Use diodes to “rectify” AC signal
- Simplest (half-wave) rectifier uses one diode:

Doing Better: Full-wave Diode Bridge

- The diode in the rectifying circuit simply prevented the negative swing of voltage from conducting
  - But this wastes half the available cycle
  - Also very irregular (bumpy): far from a “good” DC source
- By using four diodes, you can recover the negative swing:

Full-Wave Dual-Supply

- By grounding the center tap, we have two opposite AC sources
  - The diode bridge now presents + and - voltages relative to ground
  - Each can be separately smoothed/regulated
  - Cutting out diodes A and D makes a half-wave rectifier

Can buy pre-packaged diode bridges
Smoothing out the Bumps

- Still a bumpy ride, but we can smooth this out with a capacitor
  - capacitors have capacity for storing charge
  - acts like a reservoir to supply current during low spots
  - voltage regulator smoothes out remaining ripple

How smooth is smooth?

- An RC circuit has a time constant $\tau = RC$
  - because $\frac{dv}{dt} = iC$, and $i = V IR \rightarrow \frac{dv}{dt} = \frac{V}{RC}$
  - so $V = V_0 e^{(t/\tau)}$
- Any exponential function starts out with slope $= \text{Amplitude}/\tau$
- So if you want < 10% ripple over 120 Hz (8.3 ms) timescale...
  - must have $\tau = RC > 83$ ms
  - if $R = 100 \Omega$, $C > 830 \mu F$

Regulating the Voltage

- The unregulated, ripple voltage may not be at the value you want
  - depends on transformer, etc.
  - suppose you want 15.0 V
- You could use a voltage divider to set the voltage
- But it would droop under load
  - output impedance $\rightarrow R_1 || R_2$
  - need to have very small $R_1$, $R_2$ to make “stiff”
  - the divider will draw a lot of current
  - perhaps straining the source
  - power expended in divider $\gg$ power in load
- Not a “real” solution
- Important note: a “big load” means a small resistor value: 1 $\Omega$ demands more current than 1 $M\Omega$

The Zener Regulator

- Zener diodes break down at some reverse voltage
  - can buy at specific breakdown voltages
  - as long as some current goes through zener, it’ll work
  - good for rough regulation
- Conditions for working:
  - let’s maintain some minimal current, $I_z$ through zener (say a few mA)
  - then $(V_{in} - V_{out})R_1 = I_z + V_{out}R_{load}$ sets the requirement on $R_1$
  - because presumably all else is known
  - if load current increases too much, zener shuts off (node drops below breakdown) and you just have a voltage divider with the load

- high slope is what makes the zener a decent voltage regulator

- $R_1$, $R_{load}$
**Electronics Overview**

**Lecture 8**

**Winter 2012 UCSD: Physics 121; 2012**

**Electronics Overview**

**02/12/2008**

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**Voltage Regulator IC**

- Can trim down rippy voltage to precise, rock-steady value
- Now things get complicated:
  - We are now in the realm of integrated circuits (ICs)
- ICs are whole circuits in small packages
- ICs contain resistors, capacitors, diodes, transistors, etc.

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**Voltage Regulators**

- The most common voltage regulators are the LM78XX (+ voltages) and LM79XX (- voltages)
  - XX represents the voltage
    - 7815 is +15; 7915 is -15; 7805 is +5, etc.
    - Typically needs input > 3 volts above output (reg.) voltage
  - A versatile regulator is the LM317 (+) or LM337 (-)
    - 1.2–37 V output
    - \( V_{out} = 1.25(1+R_2/R_1) + I_{out}R_2 \)
    - Up to 1.5 A
    - Picture at right can go to 25 V
    - datasheetcatalog.com for details

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**Transistors**

- Transistors are versatile, highly non-linear devices
- Two frequent modes of operation:
  - Amplifiers/buffers
  - Switches
- Two main flavors:
  - npn (more common) or pnp, describing doping structure
- Also many varieties:
  - Bipolar junction transistors (BJTs) such as npn, pnp
  - Field-effect transistors (FETs): n-channel and p-channel
  - Metal-oxide-semiconductor FETs (MOSFETs)
- We'll just hit the essentials of the BJT here
  - MOSFET in later lecture

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**BJT Amplifier Mode**

- Central idea is that when in the right regime, the BJT collector-emitter current is proportional to the base current:
  - Namely, \( I_c = \beta I_b \) where \( \beta \) (sometimes \( h_{fe} \)) is typically ~100
  - In this regime, the base-emitter voltage is ~0.6 V
  - Below, \( I_b = (V_{be} - 0.6)/R_b \)
  - \( I_c = \beta I_b = \beta (V_{be} - 0.6)/R_b \)
  - So that \( V_{out} = V_{cc} - I_c R_c = V_{cc} - \beta (V_{be} - 0.6)(R_c/R_b) \)
  - Ignoring DC bias, wiggles on \( V_b \) become \( \beta (R_c/R_b) \) bigger
  - (and inverted); thus amplified

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**Beware that housing is not always ground**

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Winter 2012
Switching: Driving to Saturation

- What would happen if the base current is so big that the collector current got so big that the voltage drop across $R_c$ wants to exceed $V_{ce}$?
  - we call this saturated: $V_c - V_e$ cannot dip below ~0.2 V
  - even if $i_b$ is increased, $i_c$ won’t budge any more
- The example below is a good logic inverter
  - if $V_{cc} = 5$ V; $R_e = 1$ kΩ; $i_b$ (sat) ~ 5 mA; need $i_b$ > 0.05 mA
  - so $R_b < 20$ kΩ would put us safely into saturation if $V_{bc} = 5$ V
  - now 5 V in $\rightarrow$ ~0.2 V out; < 0.6 V in $\rightarrow$ 5 V out

Improved Zener Regulator

- By adding a transistor to the zener regulator from before, we no longer have to worry as much about the current being pulled away from the zener to the load
  - the base current is small
  - $R_{zload}$ effectively looks $\beta$ times bigger
  - real current supplied through transistor
- Can often find zeners at 5.6 V, 9.6 V, 12.6 V, 15.6 V, etc. because drop from base to emitter is about 0.6 V
  - so transistor-buffered $V_{zout}$ comes out to 5.0, 9.0, etc.
- $i_z$ varies less in this arrangement, so the regulated voltage is steadier

Transistor Buffer

- In the hookup above (emitter follower), $V_{out} = V_{in} - 0.6$
  - sounds useless, right?
  - there is no voltage "gain," but there is current gain
  - Imagine we wiggle $V_{in}$ by $\Delta V$. $V_{out}$ wiggles by the same $\Delta V$
  - so the transistor current changes by $\Delta i_e = \Delta V/R$
  - but the base current changes $1/\beta$ times this (much less)
  - so the "wiggles" think the load is $\Delta V/\Delta i_e = \beta \Delta V/i_e = \beta R$
  - the load therefore is less formidable
- The "buffer" is a way to drive a load without the driver feeling the pain (as much): it’s impedance isolation

Switching Power Supplies

- Power supplies without transformers
  - lightweight; low cost
  - can be electromagnetically noisy
- Use a DC-to-DC conversion process that relies on flipping a switch on and off, storing energy in an inductor and capacitor
  - regulators were DC-to-DC converters too, but lossy; lose $\Delta P = \Delta V \times \text{power for voltage drop of } \Delta V \text{ at current } I$
  - regulators only down-convert, but switches can also up-convert
  - switches are reasonably efficient at conversion
The FET switch is turned off or on in a pulse-width-modulation (PWM) scheme, the duty cycle of which determines the ratio of $V_{out}$ to $V_{in}$.

from: http://www.maxim-ic.com/appnotes.cfm/appnote_number/4087

**Step-down waveforms**

- Shown here is an example of the step-down with the FET duty cycle around 75%
- The average inductor current (dashed) is the current delivered to the load
  - the balance goes to the capacitor
- The ripple (parabolic sections) has peak-to-peak fractional amplitude of $\frac{2(1-D)}{8LC}$
  - so win by small $T$, large $L$ & $C$
  - $10\text{kHz}$ at $1\text{mA}$, $1000\text{µF}$ yields $\sim0.1\%$ ripple
  - means $10\text{mV}$ on $10\text{V}$

**Step-Down Calculations**

- If the FET is on for duty cycle, $D$ (fraction of time on), and the period is $T$:
  - the average output voltage is $V_{out} = DV_{in}$
  - the average current through the capacitor is zero, the average current through the load (and inductor) is $1/D$ times the input current
  - under these idealizations, power in = power out

**Cable Impedances**

- RG58 cable is characterized as $50\Omega$ cable
  - RG59 is $75\Omega$
  - some antenna cable is $300\Omega$
- Isn’t the cable nearly zero resistance? And shouldn’t the length come into play, somehow?
- There is a distinction between resistance and impedance
  - though same units
- Impedances can be real, imaginary, or complex
  - resistors are real: $Z = R$
  - capacitors and inductors are imaginary: $Z = -j\omega C$, $Z = j\omega L$
  - mixtures are complex: $Z = R - j\omega L + j\omega C$
Impedances, cont.

• Note that:
  – capacitors become less "resistive" at high frequency
  – inductors become more "resistive" at high frequency
  – bigger capacitors are more transparent
  – bigger inductors are less transparent
  – $i \cdot (\omega - 1)$ indicates 90° phase shift between voltage and current
    • after all, $V = iZ$, so $Z = V i$
    • thus if $V$ is sine wave, $i$ is cosine for inductor/capacitor
    • and given that one is derivative, one is integral, this makes sense (slide #3)
  – adding impedances automatically takes care of summation rules: add $Z$ in series
    • capacitance adds as inverse, resistors, inductors straight-up


Impedance Phasor Diagram

• Impedances can be drawn on a complex plane, with pure resistive, inductive, and capacitive impedances represented by the three cardinal arrows
• An arbitrary combination of components may have a complex impedance, which can be broken into real and imaginary parts
• Note that a system’s impedance is frequency-dependent

Transmission Line Model

• The cable has a finite capacitance per unit length
  – property of geometry and dielectric separating conductors
  – $C = \frac{2 \pi a \epsilon}{\ln(b/a)}$, where $b$ and $a$ are radii of cylinders
• Also has an inductance per unit length
  – $L = \mu (2\pi \ln(b/a))$
• When a voltage is applied, capacitors charge up
  – thus draw current; propagates down the line near speed of light
• Question: what is the ratio of voltage to current?
  – because this is the characteristic impedance
• Answer: $Z_0 = \sqrt{\frac{\mu}{\epsilon C}} = \sqrt{\frac{L}{C}} = \sqrt{(\frac{1}{2\pi}) \sqrt{\frac{\mu}{\epsilon}} \ln(b/a)}$
  – note that $Z_0$ is frequency-independent

Typical Transmission Lines

• RG58 coax is abundant
  – 30 pF per foot; 75 nH per foot; 50 $\Omega$; $v = 0.695c$, ~5 ns/m
• RG174 is the thin version
  – same parameters as above, but scaled-down geometry
• RG59
  – used for video, cable TV
  – 21 pF/ft; 118 nH per foot; 75 $\Omega$; $v = 0.695c$, ~5 ns/m
• twisted pair
  – 110 $\Omega$ at 30 turns/ft, AWG 24–28
• PCB (PC-board) trace
  – get 50 $\Omega$ if the trace width is 1.84 times the separation from the ground plane (assuming fiberglass PCB with $\varepsilon = 4.5$)
Why impedance matters

- For fast signals, get bounces (reflections) at every impedance mismatch
  - reflection amplitude is \((Z_s - Z_0)/(Z_s + Z_0)\)
  - \(s\) and \(t\) subscripts represent source and termination impedances
  - sources intending to drive a \(Z_0\) cable have \(Z_s = Z_0\)
- Consider a long cable shorted at end: insert pulse
  - driving electronics can’t know about the termination immediately; must charge up cable as the pulse propagates forward, looking like \(Z_0\) of the cable at first
  - surprise at far end: it’s a short! retreat!
  - in effect, negative pulse propagates back, nulling out capacitors (reflection is -1)
  - one round-trip later (10 ns per meter, typically), the driving electronics feels the pain of the short

Impedance matters, continued

- Now other extreme: cable un-terminated: open
  - pulse travels merely along at first, the driving electronics seeing a \(Z_0\) cable load
  - at the end, the current has nowhere to go, but driver can’t know this yet, so keeps loading cable as if it’s still \(Z_0\)
  - effectively, a positive pulse reflects back, double-charging capacitors (reflection is +1)
  - driver gets word of this one round-trip later (10 ns/m, typically), then must cease to deliver current (cable fully charged)
- The goldilocks case (reflection = 0)
  - if the end of the cable is terminated with resistor with \(R = Z_0\),
    then current is slurred up perfectly with no reflections
  - the driver is not being lied to, and hears no complaints

So Beware!

- If looking at fast (tens of ns domain) signals on scope, be sure to route signal to scope via 50 \(\Omega\) coax and terminate the scope in 50 \(\Omega\)
  - if the signal can’t drive 50 \(\Omega\), then use active probes
- Note that scope probes terminate to 1 M\(\Omega\), even though the cables are NOT 1 M\(\Omega\) cables (no such thing)
  - so scope probes can be very misleading about shapes of fast signals

References and Assignment

- References:
  - The canonical electronics reference is Horowitz and Hill: The Art of Electronics
  - Also the accompanying lab manual by Hayes and Horowitz is highly valuable (far more practically-oriented)
  - And of course: Electronics for Dogs (just ask Gromit)
- Reading
  - Sections 6.1.1, 6.1.2
  - Skim 6.2.2, 6.2.3, 6.2.4
  - Sections 6.3.1, 6.5.1, 6.5.2
  - Skim 6.3.2