Energy Scales in our Society

We'll look at the global power budget, using units of Watts and kWh. One kWh is 1000 W applied over one hour, which is 1000 J/s $\times$ 3600 s = 3.6 MJ. Also useful: 1 cal = 4.18 J; 1 Btu = 1055 J.

Disclaimer: The estimates that follow are not to be taken as definitive numbers that you can take to the bank. They are made in the spirit of the course—from guesswork and things already known. For solid data, I recommend the Annual Energy Review, put out by the Energy Information Agency.

Humans: How many kWh did you eat today?

Humans consume 1000–3000 kcal/day. We'll use 2000 kcal/day, acknowledging that this might be slightly high for a global average. 1 kcal = 4184 J $\rightarrow$ 2000 kcal $\sim$ 8.4 MJ = 2.3 kWh. Over one day (86400 s), this is about 100 W.

With 7×10^9 people (7 Gppl) on the planet, the total metabolic power of humans is 0.7 TW.

All earthly biology: A story of plankton

We have less guidance on all the bio-activity on the planet, so we'll apply the trick of the geometric mean. Clearly the ratio of all metabolic activity on earth to that of humans alone is greater than 1.0. An equally absurd limit in the other direction is perhaps 10,000. The world is a big place, but on land it is increasingly difficult to get out of sight of humans. Oceans are voluminous, but mostly dead, by volume. It's the surface area that matters, since ocean life derives from the sun—mostly in the form of plankton. And the sea surface is only three times larger than land area. Sunlight, in fact, is not the limiting factor for sea life—or we'd see opaque ocean surfaces grabbing all available sun. Nutrients limit the production rates—which is why areas of upwellings of ocean current from the ocean floor tend to be especially vibrant.

Given two extreme estimates, a and b, the geometric mean is $\lambda = \sqrt{ab}$. In this case, the "halfway" point between 10^0 and 10^4 is $10^2 = \sqrt{1 \cdot 10000}$. This would put $P_{\text{bio}} \sim 70$ TW, which is close to quoted values. Did I cheat? Well, yes. I happened to know the number already, and this may easily have guided my guess of 10,000. But I might have arrived at the same place honestly without this knowledge—something I cannot know. In any case, often the geometric mean is a nice crutch in the face of ignorance.

How efficient is the photosynthetic process? Algae top the list at 6%. A well-watered, well-fertilized Iowa cornfield turns in 1.5%. Typical numbers for plants in natural settings are 1% or below—often limited by water and/or nutrients.

Fossil Fuels

Globally, fossil fuels account for $> 80\%$ of the 15 TW power budget (37% oil; 23% coal; 22% gas). The remaining portion is split roughly equally between nuclear, hydroelectric, and biomass. Wind and solar contribute insignificant portions.

The average American probably travels 20,000 miles (30,000 km) per year in planes, trains, and automobiles. At a per-passenger average fuel economy of 40 miles per gallon (17 km/ℓ), this requires 500 gal (1900 ℓ) of gasoline per person per year, coming to about 5 ℓ per day, or about 4 kg (less dense than water).

Physicists don't know their chemistry well, and evaluating the potency of chemical reactions often involves looking up enthalpies, etc. in tables. We might try to assign 4 eV per bond, then count bonds broken and formed in a reaction to set a scale. But the sign can even be wrong in applying this approach! For instance, the octane reaction:

$$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O$$

breaks 75 bonds and creates 68—requiring a net *injection* of energy if all bonds are the same. But more generally, small differences in the strengths of different bonds could swing the balance either way. The same pathology exists for the glucose reaction (where we need to understand *structure* as well as chemical formula): more bonds are broken than formed. The missing pieces are differing bond strengths, changes in entropy, gas volume, etc. So we must resort to tables ourselves. But let's keep it simple. We're limiting ourselves to "combustion" reactions with oxygen, and for this we can use:

- 4 kcal/g for carbohydrates, proteins, and wood
- 4–8 kcal/g for coal, depending on grade
- 9 kcal/g for fats
- 10.7 kcal/g for diesel fuel
- 11.3 kcal/g for gasoline
- 13.0 kcal/g for natural gas

Some data are from the Wikipedia page on "heat of combustion." Chemistry maxes out at about 10 kcal per gram. This ultimately derives from the scale of bond strengths, at several eV. The lesser reactions above are often so because they are half-reacted already. Compare C_8H_{18} octane to $C_6H_{12}O_6$ glucose, which is already half-reacted with oxygen.

Okay, back to oil: 4 kg/day yields 4×10^4 kcal/day $\rightarrow 1.6 \times 10^8$ J/day $\rightarrow$ 2000 W per person. But this is personal transport. Let's double it to accommodate all the consumer goods shipped to a store near you (and of raw goods to the factory), plus the agricultural use of fuel for plowing, planting, harvesting, transporting. So we're at 4000 W/person $\rightarrow$ 40 humans'-worth of slave power.

Oil is about 40% of the U.S. energy use, so our total power is 10,000 W per person (>8,000 W from fossil fuels).

The U.S. uses 20% of global energy (only 5% of the world population), so 10^4 W $\cdot 3 \times 10^8$ ppl $\times 5 = 15$ TW.

Another number to keep in mind is that the U.S. uses about 20 million barrels of oil per day (about half imported). One barrel is 42 gallons, or 160 ℓ, or about 120 kg. This works out to $1/15^{\text{th}}$ of a barrel per person per day in the U.S., which translates to 8 kg—exactly in line with our numbers above.

For reference, one gallon of gasoline (3.8 ℓ) contains 36.6 kWh of combustion energy. Natural gas contains 29.3 kWh per Therm (Therm is standard billing unit: 10^5 Btu; 1 Btu = 1055 J; requires 102 cubic feet of natural gas).

Nuclear Fission

About 7% of our energy comes from nuclear fission. This relies on slow-neutron fission of ^{235}U, which makes up 0.7% of natural uranium—the rest being in ^{238}U. Natural uranium is found in an oxide called yellowcake, U_3O_8, at a density of 9 g/mℓ. For each nucleus that undergoes fission, about 200 MeV is released in all the energy channels. Considering that the ^{235}U nucleus is not far from 220 GeV in mass, the energy release is about 0.1% of the mass-energy.

Available Sunlight

The solar constant above the atmosphere is $\mathcal{F} = 1370$ W/m^2.

But what if we did not know this? Earth is in radiative equilibrium, so that $P_{\text{in}} = P_{\text{out}}$. P_{out} is via IR radiation: $P_{\text{out}} = A\sigma T^4$, and P_{in} is from the sun—mostly in the spectrum shortward of 1 μm. If the solar constant is $\mathcal{F}$, then $P_{\text{in}} = \mathcal{F}\pi R_\oplus^2 = 4\pi R_\oplus^2 \sigma T^4$. Note the factor of four geometric dilution: the sun sees cross-sectional area πR^2, while the actual surface is four times larger. The factor of four accounts for the fact that the sun is not always up, and not always straight overhead.

This gives $\mathcal{F} = 4\sigma T^4$. If we guess $T = 290$ K, and use $\sigma = 5.67 \times 10^8$ W/m^2/K^4 (remember as 5678), and note that $290^4 = (300 - 10)^4 \approx 300^4 - 4 \cdot 10 \cdot 300^3 = (300 - 40) \cdot 300^3 = 260 \cdot 27 \times 10^6 \approx 7 \times 10^9$ K^4, we get $\mathcal{F} = 4 \cdot 5.7 \cdot 7 \times 10^1 \approx 1600$ W/m^2.

1600 vs. 1370 is not *too* bad for an order-of-magnitude approach. The difference is that the greenhouse gases in our atmosphere keep the surface hotter (290 K) than the radiating outside of the "blanket," which is closer to 255 K. Also important is that 30% of the sunlight arriving at Earth is reflected directly without pausing to deposit energy. We say that earth has an *albedo* of 0.3. If we calculate: $\mathcal{F} = 4\sigma(255)^4/0.7$, we get 1370 W/m^2.

Accounting for the 30% reflection loss, Earth absorbs about 1000 W per square meter of projected area: a convenient number. So the sunlight absorbed is $\pi R_\oplus^2 \times 1000$ W/m^2. $R_\oplus = 6.4 \times 10^6$ m; $R_\oplus^2 \approx 4 \times 10^{13}$ m^2 $\rightarrow P_{\text{abs}} \approx 1.2 \times 10^{17}$ W, or 120,000 TW.

This is:

- 2,000 times bigger than earth's biology
- 10,000 times bigger than our industrial energy budget (fossil fuels)
- 200,000 times bigger than human metabolism

So compared to the sun, all the rest is mere crumbs!

So We Keep Going, Then?

For the last few centuries, we have increased our global energy usage (now at $\sim$15 TW) at a rate of about $r = 3\%$ per year. How many years to double? $(1 + r)^y = 2 \rightarrow y = \ln 2/\ln(1 + r)$. Since $r \ll 1$, $y \approx \ln 2/r \approx 0.69/r \rightarrow 23$ years to double at 3% growth ["law of 70": doubles in 70/%-rate years]. We see a ten-fold increase in 76 years at 3%. Let's make our math easier and claim the characteristic rate is 2.3%, so that we increase our scale one order of magnitude every 100 years.

If we used 100% of the land area (25% of globe) to convert incident sunlight into industrial energy at 40% efficiency, we could expand to 1000 times bigger than our current scale, taking 300 years.

What if we used 100% of the earth surface for energy at 100% efficiency (thermodynamically impossible)? We buy another 100 years. The year is 2410 and we're out of juice.

Still hungry? If we surround the sun entirely (a Dyson sphere) and use the energy at 100% efficiency, we gain a factor of $4\pi r_{\text{AU}}^2/\pi R_\oplus^2 = 4 \cdot (1.5 \times 10^{11}\,\text{m})^2/(6.4 \times 10^6\,\text{m})^2 \approx 9 \times 10^{22}/4 \times 10^{13} \sim 2 \times 10^9$, taking 935 years longer, 1335 years from now.

But wait—there's more! Our sun isn't the only star in the Galaxy. There are about 10^{11} more. If we use each one at 100% efficiency, we'll buy another 1100 years.

So at 2.3% growth rate in energy use, we exhaust the Galaxy's stellar supply in 2500 years! This is a historical timescale.

Clear Absurdity

What's absurd about this scenario? Oh boy—where to start?

- The 3% growth seen in the last few centuries is anomalous over our history: fossil fuels represent a one-time binge. We found the earth's battery, and hooked up Las Vegas (effectively a short-circuit) before stopping to think.

- If population saturates or turns downward, continued growth on this trajectory is unlikely. Flipping this around, because continued physical growth is absurd, population will have *no choice* but to saturate or turn down.

- Where would the materials come from to surround the sun with a shell at 1 AU? The volume of such a shell with thickness Δr would be $4\pi r_{\mathrm{AU}}^2 \Delta r$. If we use the Earth as our feedstock, we find that $\Delta r = \frac{1}{3}\frac{R_\oplus^3}{r_{\mathrm{AU}}^2} = \frac{1}{3}\frac{2.6\times10^{20}\,\mathrm{m}^3}{2.3\times10^{22}\,\mathrm{m}^2} \approx 3$ mm. But this is 3 mm of rock/iron. We'd really like photovoltaics or something. Not to mention that a rigid shell this thin could not support itself against solar gravity: even if spinning the poles will be crushed.

Thermodynamic Dimensions

Another way to look at limitations to growth: any energy we generate here on the planet—even by fusion or a futuristic energy to be discovered—the inevitable energy cascade results in heat. The only way off the planet is via radiation: $P = A_\oplus \sigma T^4$. As noted above, Earth absorbes solar power at a rate of about 120,000 TW, which is approximately 10,000 times our 15 TW energy budget. But at 2.3% per year growth, this leads to just 400 years of overhead. Therefore on this timescale, we would double the power dissipation on Earth, resulting in a temperature increase by a factor of $2^{\frac{1}{4}} \approx 1.2$, meaning 360 K instead of 300 K. This makes the 33 K total Greenhouse effect a small contribution, let alone the few-degree modification associated with anthropogenic climate change. So we can't go there. On the timescale of a few centuries, we must halt energy growth on this planet.

The lesson is that the growth we've experienced for many generations is necessarily transitory, and does not define our future. One way or another, the narrative must change. Lest you think fusion saves us, bear in mind that by 1300 years from now, we'd be outstripping the sun. Constant growth might be the mantra of our economists, but we physicists can say it's not part of our real future.

Fermi Approach Solves Fermi Problem?

This Fermi approach to problem solving may hold a clue to the Fermi Paradox. This is based on another Fermi-type problem: how many alien civilizations should we be able to detect? This is embodied in the famous Drake Equation:

$$N_{\mathrm{detect}} = N_{\mathrm{stars}} f_{\mathrm{planets}} f_{\mathrm{habitable}} f_{\mathrm{life}} f_{\mathrm{civilization}} f_{\mathrm{detectable}} \eta_{\mathrm{survive}},$$

which is just a long product getting at the fraction of stars hosting civilized life, together with the chance that they broadcast detectable signals, and finally a duty cycle for duration of the civilization. In our galaxy, we have 10^{11} stars, at *least* 10% of which have planets (evidence points to more), and of these, at least 10% are in the habitable zone (liquid water). Maybe only 1% of these will cook up life (wild variation on this estimate), and maybe 1% of these give rise to civilization. Perhaps only 0.1% would be detectable to us (distance one factor), and perhaps they can figure out how to survive for a tenth the age of the Universe ($\eta_{\mathrm{survive}} \sim 0.1$). These combine to 10 detectable civilizations today. Many enthusiasts put the numbers higher. Fermi's paradox was simply: given the presumably favorable numbers, why no signs?

The section above highlighted the necessarily temporary nature of growth. Our technological phase is inseparable from fossil fuel use, which is finite, and lasts 200 years (half-way through now). What if we don't continue high-tech after fossil fuels? After all, evolution is incremental, and we are the first increment smart enough to find/use fossil fuels. Maybe we're not smart enough *not* to use fossil fuels. Why would evolution skip steps and make us *wise*? Maybe this is a common phenomenon: planets with life have fossil energy reserves and the first ones to find/exploit them run themselves into the ground utterly dependent on the fading resource. Evolution and its incrementalism is the common thread. This may put $\eta_{\mathrm{survive}} < (1000\,\mathrm{yr}/10\,\mathrm{Gyr}) \lesssim 10^{-7}$, in which case there is no paradox. If this is right, then we're likely the only technological civilization (e.g., with consumer electronics) in the galaxy right now. Cool, huh?