Optical Phenomena in Nature

A Feast for the Eyes

So many phenomena, so little time—and where's the camera when you need it? The natural world is full of amazing sights that the untrained eye might easily overlook. Not everyone gets a kick out of spotting these treats, but I think physicists—of all people—may be particularly drawn to these funky observations. This is not the place to expound on the rich variety of topics. For this, I recommend the books listed at the end of these lecture notes. For now, as a teaser, I just list some of the things to keep an eye out for.

- the $22°$ halo (around moon or sun; equally likely, though solar halo less often noticed!)
- sun dogs, or parhelia—same ice crystals that make the halo, but settled in stable air
- pillars (usually above or below sun)
- the sub-sun (when above clouds); same circumstance that makes sun dogs
- sub-sun dogs—yes I have seen sun dogs sourced by the sub-sun!!
- glories: several color cycles around anti-solar point—look for airplane shadow in center (and can sometimes tell where in plane you sit, like a bulls-eye on your position)
- shadow-hiding, or heiligenschein—seen on grassy lawn or from airplane as halo around anti-solar point
- earth shadow projected onto twilight sky
- the green flash—often visible in Hawaii, but best I've seen was from glider port in La Jolla
- secondary rainbow (and note darkness inside primary rainbow)

The list can continue, and indeed can form enough material for excellent books.

Heisenberg at your Fingertips

We think of quantum-mechanical phenomena as being beyond the limits of our macroscopic existence. Yet simply bringing your thumb and forefinger into a nearly-touching arrangement makes a dark liquid-like feature before they physically touch. Isn't this just wave behavior? Well, in the particle interpretation, we know that we have confined the photon to a vertical space Δx between the thumb and forefinger. We know that the resulting vertical momentum of the photon must be uncertain at the level of about $\Delta p \sim \hbar/\Delta x$. Meanwhile, the forward momentum of the photon is $h\nu/c = h/\lambda$. So the angular uncertainty imposed on the photon's path is $\Delta\theta \sim \Delta p/p = \hbar\lambda/h\Delta x \sim \lambda/\Delta x$, casually discarding numerical factors. This looks just like the familiar $\Delta\theta \sim \lambda/D$ diffraction. One could argue that the h and $\hbar$ canceled each other, so we really haven't probed the quantum world. But it's because we asked about the photon *angle*, and not absolute momentum, that we get this cancellation. Of course we could argue particle-wave duality all over again, but I'm satisfied that in the particle context, diffraction is just the uncertainty principle. If still unconvinced, switch from thinking about photons to electrons, which also exhibit diffractive spread when passed through a narrow slit.

But diffraction shows up in more places than just this. A partial list:

- if you squint, at first things may get sharp (pinhole effect), then get blurry as diffraction kicks in

- also when squinting you may often get a spray of light diffracted from an edge
- radial streaks in windshield, CD/DVD are from diffraction off grooves from wipers/pits
- carry a pinhole around and experience the world in diffracted terms
- sometimes light through slits in blinds show fringes—especially if the illumination is from a angularly compact glint from a car

Refractive Index of Air

Can we understand why the refractive index of air is what it is—around $n_{\rm air} \approx 1.00028$? I had some trouble thinking about it, but Tom O'Neil deftly took me down the E&M path to understanding. The key issue is the polarization of air molecules, meaning the displacement of the electron cloud in the presence of an electric field, $\mathbf{E}$. The interaction of light with molecules results in a permittivity, ε, greater than that of free-space (ε_0). Because the speed of light is $c = (\mu_0\varepsilon_0)^{-\frac{1}{2}}$, the group velocity of light in a dielectric medium becomes $v = (\mu_0\varepsilon)^{-\frac{1}{2}} \equiv c/n$, assuming the medium does not change the magnetic permeability. This means $n = \sqrt{\varepsilon/\varepsilon_0} = \sqrt{\varepsilon_{\rm r}}$.

In the presence of polarizing elements, we define the *electric displacement field*,

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P},$$

where $\mathbf{P}$ is the *polarization* of the medium, having units of C/m^2 (see Lecture 4 for an electromagnetic units extravaganza in the SI system). We can think of $\mathbf{P}$ as a dipole density—individual dipoles defined by $\mathbf{p} = e\Delta\mathbf{r}$, in C·m. If we call the number density of atoms/molecules η (would normally use n, but avoiding confusion with refractive index), the total polarization is just $\mathbf{P} = \eta\mathbf{p}$.

If we subject an atom (here a hydrogen atom for simple thinking) to an external electric field, the resulting force, $e\mathbf{E}$, competes against the central force, $\mathbf{F}_{\rm c}$, to produce a displacement $\Delta\mathbf{r}$. The force required to maintain displacement $\Delta\mathbf{r}$ against $\mathbf{F}_{\rm c}$ is $\Delta\mathbf{F} = \mathbf{F}'_{\rm c}\Delta\mathbf{r}$. We therefore get:

$$\Delta\mathbf{F} = \frac{2e^2}{4\pi\varepsilon_0 r^3}\Delta\mathbf{r} = e\mathbf{E}.$$

Dividing both sides by e and recognizing the useful nugget $\mathbf{p} = e\Delta\mathbf{r}$ on the left side, we get that

$$\mathbf{p} = e\Delta\mathbf{r} = 2\pi\varepsilon_0 r^3\mathbf{E}.$$

Now we can put pieces back together to see that

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E}(1 + 2\pi\eta r^3),$$

so we can say $\varepsilon/\varepsilon_0 = 1 + 2\pi\eta r^3$, making $n = \sqrt{\varepsilon/\varepsilon_0} \approx 1 + \pi\eta a^3$, where we have assumed a refractive index near unity, and replaced r with a as the representative radius of the atom/molecule. The quantity ηa^3 is something like a volumetric filling factor of the medium, which is indeed small for a gas. We have calculated before in the class that $\eta = P/kT \sim 2.7 \times 10^{25}$ m^{-3}. If we use $a = 10^{-10}$ m, we find that $N \equiv (n-1) \times 10^6 \sim 80$, compared to the target value of $N = 280$. We can quickly fix this by assigning $a = 1.5$ Å, which seems perfectly reasonable for an N_2 or O_2 molecule. But note that we never departed from our hydrogen atom in describing the polarization.

The main point is to see that we can understand something of the basis for the magnitude of the refractive index of air. Also very important is to notice that the *refractive index is proportional to the density* of the medium. Since air density changes according to temperature and pressure, we can write that

$$N \equiv (n-1) \times 10^6 = 0.790\frac{P}{T}, \text{ at } \lambda = 550\,\text{nm},$$

where P is in Pa and T is in Kelvin. Humidity changes $(n-1)$ in the visible band by at most 0.3% at 20°C. Water is a far more significant influence in microwave bands (resonance phenomenon).

The refractive index is a weak function of wavelength, becoming stronger toward the ultraviolet as a resonance is approached. At the average earth surface temperature of 288 K (15°C), we get the following values for N:

λ (nm)	$N \equiv (n-1) \times 10^6$
350	286
400	283
550	278
700	276
1000	274
∞	272.7

Over the visible band, N changes by $\frac{7}{280} = \frac{1}{40}$, or 2.5%.

Refractive Delay in Atmosphere

If one shoots a laser pulse up through the atmosphere, how delayed will it be by the time it emerges from the atmosphere, relative to a pretend pulse that travels through vacuum instead? Need we worry about the refractive index profile as we ascend through the atmosphere? Each layer of the atmosphere of thickness dz imposes a relative delay of $(n(z)-1)dz$. But since $n-1$ is just proportional to density, the vertical integral amounts to an integral of density, yielding simply the total column of air overhead. So we are safe using just the scale height, $h \sim 8$ km. The total delay will then be $\Delta s = (n-1)h \approx 280 \times 10^{-6} \cdot 8 \times 10^3 \sim 2$ m.

Note that the integrated number density, $\int \eta dz$ overhead is proportional to atmospheric pressure, as pressure is just $P = \int \rho g dz$, and η is related to ρ by the mass of the associated molecule. So a measure of surface pressure is an excellent proxy to the total refractive delay at that time.

Mirages

Hot roads often show mirages at grazing-incidence angles. When can this happen? If we consider breaking the air into infinitesimal vertical layers, each with its own refractive index, n_i—presumably but not necessarily changing in some sensible pattern—we can describe the deflection of a ray passing from one layer to the next with Snell's law:

$$n_i \sin\theta_i = n_{i+1} \sin\theta_{i+1},$$

where θ is the incidence angle relative to the (here vertical) surface normal (hence near $\pi/2$ for mirage geometries). We can imagine following a ray through layer after layer, each time obeying the above relation, soon realizing that

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_i \sin\theta_i.$$

No matter what step we are in, we can relate our current value of $n \sin\theta$ to the initial values. In other words, $n \sin\theta = \text{const.} = n_0 \sin\theta_0$.

A mirage will happen if the light ray is prohibited from reaching the ground, arriving at a path parallel to the surface, or $\theta_{\rm f} = \frac{\pi}{2}$, so that $\sin\theta_{\rm f} = 1$, meaning $\sin\theta_0 = n_{\rm f}/n_0$. Because we are dealing with grazing angles, θ_0 is near $\frac{\pi}{2}$, and we might more intuitively switch to $\delta = \frac{\pi}{2} - \theta$, so $\cos\delta = n_{\rm f}/n_0 \approx 1 - \frac{1}{2}\delta^2$. We can rewrite this as $\delta^2 = 2(1 - \frac{n_{\rm f}}{n_0}) = 2\Delta n/n_0$, with $\Delta n = n_0 - n_{\rm f}$. Since n_0 is so very nearly unity, we can blithely ignore its influence in the relation, so we get $\delta = \sqrt{2\Delta n}$.

How much does the refractive index have to change (thus how much does the air temperature have to change) to permit a mirage at a particular angle, δ? Since $n - 1 \approx 0.79 \times 10^{-6}\frac{P}{T}$, we have that $\Delta n = (n-1)\frac{\Delta T}{T}$ (we can arrive at this by constructing derivatives: $\frac{\Delta n}{\Delta T} = 0.79 \times 10^{-6}\frac{P}{T^2} = \frac{n-1}{T}$). Numerically, this is nice because $n - 1 \approx 280 \times 10^{-6}$ and we can pick $T \sim 280$ K, so that $\delta = \sqrt{2\Delta n} = \sqrt{2 \times 10^{-6}\Delta T} = 1.4 \times 10^{-3}\sqrt{\Delta T}$. In order to get a mirage at an sensible angle of 0.5° (angular diameter of sun or moon), then we need

$\delta \sim \frac{1}{120} \approx 0.008$, requiring $\sqrt{\Delta T} \sim 6$, or $\Delta T \sim 36$ K. This seems reasonable for a hot road. In the absence of convection, a black surface in full sun will achieve a radiative equilibrium at a whopping 90°C. Get out the egg.

Getting Stuck and Refractive Child's Play

In the scheme above, we used the condition that the final angle, θ_f went to $\frac{\pi}{2}$ (horizontal). Once the ray is horizontal, why doesn't it *stay* that way? What would encourage it to begin an upward journey? The answer is that light *rays* are unphysical, albeit useful tools for understanding preferred paths. But the light itself has extent, and "explores" nearby options. In doing so, it will "sense" that the refractive index is higher above it, and will curve into this medium.

It turns out you can think of light as being like two wheels on an axle, like a dismantled child's wagon or maybe a Lego construct. If different media have different natural speeds for the wheel (e.g., fast on sidewalk, slow in grass), the wheel-set will deflect its direction when it encounters a change in medium at some non-normal angle of incidence. It can be shown that this reproduces Snell's law exactly, and can even get total internal reflection right. In the case of the mirage, at the bottom of the ray path (horizontal), the "lower wheel" is moving faster than the "upper wheel" due to the lower refractive index near the hot road. So it will continue to change its angle, even when the mathematical ray has no incentive to do so.

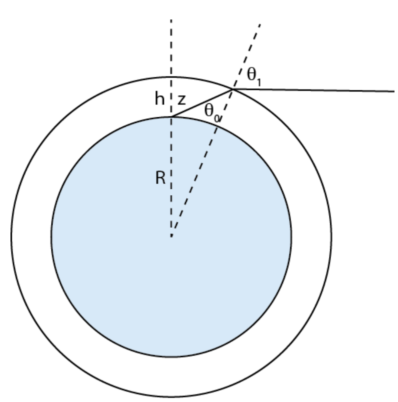
I the real world, there will be turbulence and fluctuations that also may jog the ray off its horizontal path. But these are not necessary for light to curve back upward. The wave nature in conjunction with a gradient will make sure of it.

Refraction by the Atmosphere

When we see the sun first touch the ocean horizon, the sun has "actually" already set. That is, without atmospheric refraction the sun would already be wholly (but *just*) below the horizon. The curved path of light through the atmosphere "lifts" the sun up by over half-a degree. Even at high elevation angles, the atmospheric refraction is significant for astronomical telescopes, which are required to point and track objects at the 1 arcsecond (5×10^{-6} rad) level.

Step Function

If we take the crudest possible model for the atmosphere we do surprisingly well at describing the refractive effect. We will enforce a uniform atmosphere that is one scale height, h, thick at a constant refractive index n_air. The ray leaves the observer with zenith angle z and radius R, arriving at the top of our atmosphere at incidence angle θ_0 and radius $R + h$. The law of sines tells us that $\sin\theta_0 = \frac{R}{R+h}\sin z$, and Snell's law gives us $n \sin\theta_0 = \sin\theta_1$.



We are interested in $\Delta\theta = \theta_1 - \theta_0$. Note that $\sin\theta_1 = \sin(\theta_0 + \Delta\theta) \approx \sin\theta_0 + \Delta\theta\cos\theta_0$ for small $\Delta\theta$. Putting these together, we have $(n-1)\sin\theta_0 = \Delta\theta\cos\theta_0$, so $\Delta\theta = (n-1)\tan\theta_0$. We can then work to get $\tan\theta_0$ from the relation above: $\sin\theta_0 = \frac{R}{R+h}\sin z$. Doing so yields:

$$\tan\theta_0 = \frac{\sin\theta_0}{\sqrt{1-\sin^2\theta_0}},$$

$$\Delta\theta = (n-1)\frac{R\sin z}{\sqrt{(R+h)^2 - R^2\sin^2 z}}.$$

For modest zenith angles, the ratio $\frac{R}{R+h}$ is so nearly unity that $\theta_0 \approx z$, and we can say $\Delta\theta \approx (n-1)\tan z$. At 45°, this results in $\Delta\theta \sim 280 \times 10^{-6}$, or about one arcminute.

On the horizon, $\sin z = 1$, and we get $\Delta\theta \approx (n-1)\sqrt{\frac{R}{2h}}$, which works out to about 20 arcminutes, or one third of a degree. Not quite the just-over-half degree promised above, but not too far off for such a bone-headed model. Comparing to an online calculator, here is the comparison, with deflections presented in arcminutes (works pretty darned well even to 5° off the horizon):

zenith angle, z	Step (arcmin)	Continuous (arcmin)	"real" (arcmin)
45°	0.94	0.94	0.94
80°	5.13	5.15	5.14
85°	9.33	9.46	9.53
89.5°	18.5	19.6	27.7
90°	18.8	20.0	33.2

Continuous Model

The continuous model in the table above is one in which the atmosphere is allowed to have a gradient, and the value $nr\sin\theta$ is preserved, much like the constant in the mirage development above. Here, r is the distance to the spherical coordinate origin (center of the earth). But be aware that θ is the angle relative to the curving arcs of atmosphere (angle of incidence at each layer), so one must compare the final θ to an undeflected θ, which itself maintains constant $r\sin\theta$, acting as a sort of impact parameter. This model results in:

$$\Delta\theta = \sin^{-1}\left[\frac{n_0 R\sin z}{R+h}\right] - \sin^{-1}\left[\frac{R\sin z}{R+h}\right].$$

The reason 89.5° is included in the table is that this is approximately where the top of the sun is when the bottom touches the horizon. Note that the differential refraction is about 5.5 arcminutes, or almost 20% of the sun's 30 arcminute diameter. So the sun appears squashed when it is close to the horizon: the bottom of the sun is being scrunched up more dramatically than is the top.

Dispersion and the Green Flash

We saw in the section on the index of refraction that the refractive index is a weak function of wavelength. Over the visible-light band, it is a 2.5% effect. This actually stretches out stars in the vertical direction, with the blue image of the star appearing highest in the sky, and the red image lowest. Astronomical observations tend to be narrow-band enough that this effect is reduced. But spectroscopy covering the visible band must take special care to orient the slit vertically on the sky so that the light admitted is not an unintended function of wavelength—imprinting an undesired spectral shape onto the data. A star at 45° experiences a roughly 60 arcsecond refractive effect, which is large compared to its roughly arcsecond size. The red-to-blue smear is 1.5 arcseconds (2.5% of total refraction), so is already problematic. It only gets worse as one approaches the horizon.

This phenomenon also means that when the sun is on the horizon, being refracted upward by about one solar diameter, the image actually breaks into a smear about 2.5% of the height of the sun. In other words, the red sun is lowest, and the blue sun the highest. The red sun sets first, then the green sun, then the blue sun.

But there is very little blue light left in the sunlight traveling through the equivalent of about 40 atmospheres along the horizon—being preferentially scattered out according to the λ^{-4} scaling. Yet there may be some green left, if the sun is not a deep red or orange on the horizon (requires clear air). Since the green sun sets last, this is the last part you see. It is called a green flash, even though it's more of a slow wink of green in the last second or two of the sunset. Care must be taken not to saturate your eyes by staring at the sun prematurely. Ocean horizons are ideal. I have seen many from Hawaii, for instance—but the best I have seen was from the Glider Port in La Jolla.

How long should we expect the green flash to last? The sun moves about 360° in 24 h, or 15° per hour, taking 2 min to cross its own diameter (thus sunset is about a two minute affair). It will take 1% of this two minutes to run through the wavelength spread (1% from red to green: not quite the whole 2.5% across the visible spectrum), or about 1 second. This time is tempered by the geometry of the sun's encounter with the horizon. At the north or south pole, the sunrise takes a day as the sun slides through its declination range, and this greatly extends your chances of seeing a green "flash"—and your eyes will be frozen open anyway.

References

Minnaert, T*he Nature of Light and Colour in the Open Air*; a classic text from a truly keen observer.

Lynch and Livingston, *Color and Light in Nature*; inspired by Minnaert, but modernized with many excellent color photos and reasonable depth of discussion of the related phenomena.