Springs and Things

Restoring Force
Oscillation and Resonance
Model for Molecules

Springy Things

Example

• If the springs in your 1000 kg car compress by 10 cm (e.g., when lowered off of jacks):
  – then the springs must be exerting \( mg = 10,000 \) Newtons of force to support the car
  – \( F = -k\Delta x = 10,000 \text{ N, } \Delta x = -0.1 \text{ m} \)
  – so \( k = 100,000 \text{ N/m (stiff spring)} \)
    • this is the collective spring constant: they all add to this
  • Now if you pile 400 kg into your car, how much will it sink?
    – \( 4,000 = (100,000)\Delta x \), so \( \Delta x = 4/100 = 0.04 \text{ m} = 4 \text{ cm} \)
  • Could have taken short-cut:
    – springs are linear, so 400 additional kg will depress car an additional 40% (400/1000) of its initial depression

Springs: supplying restoring force

Energy Storage in Spring

• Applied force is \( k\Delta x \) (reaction from spring is \(-k\Delta x\))
  – starts at zero when \( \Delta x = 0 \)
  – slowly ramps up as you push
• Work is force times distance
  • Let’s say we want to move spring a total distance of \( \Delta x \)
    – would naively think \( W = k\Delta x^2 \)
    – but force starts out small (not full \( k\Delta x \) right away)
    – works out that \( W = \frac{1}{2}k\Delta x^2 \)
Work “Integral”

• Since work is force times distance, and the force ramps up as we compress the spring further...
  - takes more work (area of rectangle) to compress a little bit more (width of rectangle) as force increases (height of rectangle)
  - if full distance compressed is \( k \Delta x \), then force is \( k \Delta x \), and area under force “curve” is \( \frac{1}{2} (base)(height) = \frac{1}{2} (\Delta x)k\Delta x = \frac{1}{2}k \Delta x^2 \)
  - area under curve is called an integral: work is integral of force

The Potential Energy Function

• Since the potential energy varies with the square of displacement, we can plot this as a parabola
• Call the low point zero potential
• Think of it like the drawing of a trough between two hillsides
• A ball would roll back and forth exchanging gravitational potential for kinetic energy
• Likewise, a compressed (or stretched) spring and mass combination will oscillate
  - exchanges kinetic energy for potential energy of spring

Example of Oscillation

• Plot shows position (displacement) on the vertical axis and time on the horizontal axis
• Oscillation is clear
• Damping is present (amplitude decreases)
  - envelope is decaying exponential function

Frequency of Oscillation

• Mass will execute some number of cycles per second (could be less than one)
• This is the frequency of oscillation (measured in Hertz, or cycles per second)
• The frequency is proportional to the square root of the spring constant divided by the mass:
  \[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]
• Larger mass means more sluggish (lower freq.)
• Larger (stiffer) spring constant means faster (higher freq.)
Natural Frequencies & Damping

- Many physical systems exhibit oscillation
  - guitar strings, piano strings, violin strings, air in flute
  - lampposts, trees, rulers hung off edge of table
  - buildings, bridges, parking structures

- Some are “cleaner” than others
  - depends on complexity of system: how many natural frequencies exist
    - a tree has many: many branches of different sizes
    - damping: energy loss mechanisms (friction, radiation)
      - a tree has a lot of damping from air resistance
      - cars have “shocks” (shock absorbers) to absorb oscillation energy
  - elastic is a word used to describe lossless (or nearly so) systems
    - “bouncy” also gets at the right idea

Resonance

- If you apply a periodic force to a system at or near its natural frequency, it may resonate
  - depends on how closely the frequency matches
    - damping limits resonance

- Driving below the frequency, it deflects with the force

- Driving above the frequency, it doesn’t do much at all

- Picture below shows amplitude of response oscillation when driving force changes frequency

Resonance Examples

- Shattering wine glass
  - if “pumped” at natural frequency, amplitude builds up until it shatters

- Swinging on swingset
  - you learn to “pump” at natural frequency of swing
  - amplitude of swing builds up

- Tacoma Narrows Bridge
  - eddies of wind shedding of top and bottom of bridge in alternating fashion “pumped” bridge at natural oscillation frequency
  - totally shattered
  - big lesson for today’s bridge builders: include damping

Wiggling Molecules/Crystals

- Now imagine models of molecules built out of spring connections
- Result is very wiggly
- Thermal energy (heat content) manifests itself as incessant wiggling of the atoms composing molecules and crystals (solids)
- This will be important in discussing:
  - microwave ovens
  - colors of materials
  - optical properties
  - heat conduction
A model for crystals/molecules

- We can think of molecules as masses connected by springs.
- Even neutral atoms attract when they are close, but repel when they get too close.
  - Electrons “see” (and like/covet) the neighboring nucleus.
  - But when the electrons start to overlap, repulsion takes over.
  - Try moving in with the neighbor you covet!
- The trough looks just like the spring potential.
  - So the “connection” is spring-like.

Estimation: How fast do they wiggle?

- A 1 kg block of wood takes 1000 J to heat by 1 °C.
  - Just a restatement of heat capacity = 1000 J/°C.
  - So from 0 to 300 K, it takes 300,000 J.
- If we assign some kinetic energy to each mass (atom), it must all add up to 300,000 J.
- The velocities are randomly oriented, but we can still say that\[ \frac{1}{2}mv^2 = 300,000 \text{ J} \]
  - So \[ v^2 = 600,000 \text{ (m/s)}^2 \]
  - Characteristic \( v \approx 800 \text{ m/s} \) (very fast!).
- This is in the right ballpark for the velocities of atoms buzzing about within materials at room temperature.
  - It’s what we mean by heat.

Assignments

- HW1 due today.
- First bi-weekly question/observation due tomorrow (4/14).
  - 6PM cutoff is strict; half credit for following week.