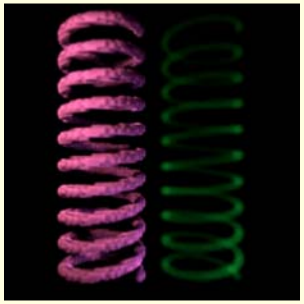
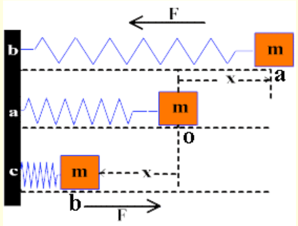


Springy Things

Restoring Force
Oscillation and Resonance
Model for Molecules

Springs: supplying restoring force



- When you pull on (stretch) a spring, it pulls back (top picture)
- When you push on (compress) a spring, it pushes back (bottom)
- Thus springs present a *restoring* force:

  $F = -k\Delta x$

  - Δx is the displacement (in meters)
  - k is the "spring constant" in Newtons per meter (N/m)
  - the negative sign means opposite to the direction of displacement

Example

- If the springs in your 1000 kg car compress by 10 cm (e.g., when lowered off of jacks):
  - then the springs must be exerting mg = 10,000 Newtons of force to support the car
  - $F = -k\Delta x$ = 10,000 N, Δx = –0.1 m
  - so k = 100,000 N/m (stiff spring)
    - this is the collective spring constant: they all add to this
- Now if you pile 400 kg into your car, how much will it sink?
  - 4,000 = (100,000)Δx, so Δx = 4/100 = 0.04 m = 4 cm
- Could have taken short-cut:
  - springs are *linear*, so 400 additional kg will depress car an additional 40% (400/1000) of its initial depression

Energy Storage in Spring

- Applied force is $k\Delta x$ (reaction from spring is $-k\Delta x$)
  - starts at zero when $\Delta x = 0$
  - slowly ramps up as you push
- Work is force times distance
- Let's say we want to move spring a total distance of Δx
  - would naively think $W = k\Delta x^2$
  - but force starts out small (not full $k\Delta x$ right away)
  - works out that $W = ½k\Delta x^2$

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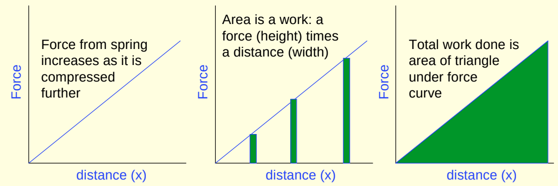
Work "Integral"

- Since work is force times distance, and the force ramps up as we compress the spring further…



  - takes more work (area of rectangle) to compress a little bit more (width of rectangle) as force increases (height of rectangle)
  - if full distance compressed is $k\Delta x$, then force is $k\Delta x$, and area under force "curve" is ½(base)(height) = ½(Δx)$k\Delta x$ = ½$k\Delta x^2$
  - area under curve is called an integral: work is integral of force

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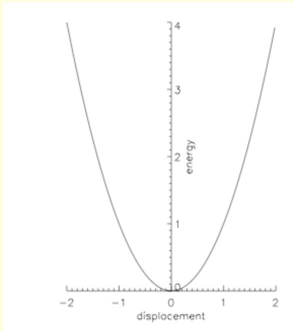
The Potential Energy Function

- Since the potential energy varies with the square of displacement, we can plot this as a parabola
- Call the low point zero potential
- Think of it like the drawing of a trough between two hillsides
- A ball would roll back and forth exchanging gravitational potential for kinetic energy
- Likewise, a compressed (or stretched) spring and mass combination will oscillate
  - exchanges kinetic energy for potential energy of spring



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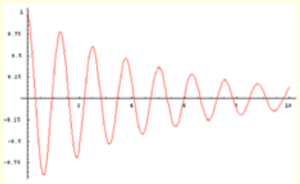
Example of Oscillation

- Plot shows position (displacement) on the vertical axis and time on the horizontal axis
- Oscillation is clear
- Damping is present (amplitude decreases)
  - envelope is decaying exponential function



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Frequency of Oscillation

- Mass will execute some number of cycles per second (could be less than one)
- This is the frequency of oscillation (measured in Hertz, or cycles per second)
- The frequency is proportional to the square root of the spring constant divided by the mass:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- Larger mass means more sluggish (lower freq.)
- Larger (stiffer) spring constant means faster (higher freq.)

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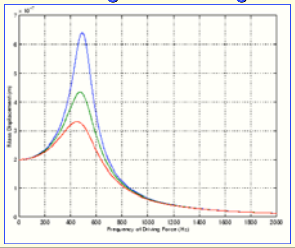
Natural Frequencies & Damping

- Many physical systems exhibit oscillation
  - guitar strings, piano strings, violin strings, air in flute
  - lampposts, trees, rulers hung off edge of table
  - buildings, bridges, parking structures
- Some are "cleaner" than others
  - depends on complexity of system: how many natural frequencies exist
    - a tree has many: many branches of different sizes
  - damping: energy loss mechanisms (friction, radiation)
    - a tree has a lot of damping from air resistance
    - cars have "shocks" (shock absorbers) to absorb oscillation energy
  - elastic is a word used to describe lossless (or nearly so) systems
    - "bouncy" also gets at the right idea

Resonance

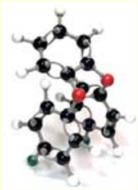
- If you apply a periodic force to a system at or near its natural frequency, it may resonate
  - depends on how closely the frequency matches
  - damping limits resonance
- Driving below the frequency, it deflects with the force
- Driving above the frequency, it doesn't do much at all
- Picture below shows amplitude of response oscillation when driving force changes frequency

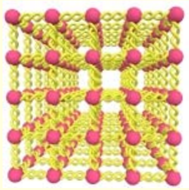


Resonance Examples

- Shattering wine glass
  - if "pumped" at natural frequency, amplitude builds up until it shatters
- Swinging on swingset
  - you learn to "pump" at natural frequency of swing
  - amplitude of swing builds up
- Tacoma Narrows Bridge
  - eddies of wind shedding of top and bottom of bridge in alternating fashion "pumped" bridge at natural oscillation frequency
  - totally shattered
  - big lesson for today's bridge builders: include damping

Wiggling Molecules/Crystals

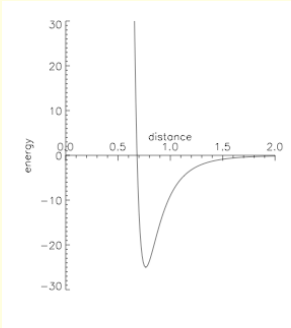




- Now imagine models of molecules built out of spring connections
- Result is very wiggly
- Thermal energy (heat content) manifests itself as incessant wiggling of the atoms composing molecules and crystals (solids)
- This will be important in discussing:
  - microwave ovens
  - colors of materials
  - optical properties
  - heat conduction

A model for crystals/molecules

- We can think of molecules as masses connected by springs
- Even neutral atoms attract when they are close, but repel when they get *too* close
  - electrons "see" (and like/covet) the neighboring nucleus
  - but when the electrons start to overlap, repulsion takes over
    - try moving in with the neighbor you covet!
- The trough looks just like the spring potential
  - so the "connection" is spring-like



Estimation: How fast do they wiggle?

- A 1 kg block of wood takes 1000 J to heat by 1 °C
  - just a restatement of heat capacity = 1000 J/kg/°C
  - so from 0 to 300 K, it takes 300,000 J
- If we assign some kinetic energy to each mass (atom), it must all add up to 300,000 J
- The velocities are randomly oriented, but we can still say that $\frac{1}{2}mv^2 = 300{,}000$ J
  - so $v^2 = 600{,}000\ (\mathrm{m/s})^2$
  - characteristic $v = 800$ m/s (very fast!)
- This is in the right ballpark for the velocities of atoms buzzing about within materials at room temperature
  - it's what we *mean* by *heat*

Assignments

- HW1 due today
- First bi-weekly question/observation due tomorrow (4/14)
  - 6PM cutoff is strict; half credit for following week
- Reading: Chapter 10: 302–308, 324–330 for Tuesday
- HW2: 7.E.1, 7.E.4, 7.P.1, 7.P.2, 7.P.3, 3.P.2, 3.P.4, plus eight additional *required* problems available on assignments page