APOLLO Lunar Prediction

August 28, 2005

1 Introduction

At its highest resolution, the time-to-digital converter (TDC) used in the APOLLO timing system has a "window" only 100 ns in duration. This corresponds to 15 m in one-way range. This means that we must predict the round-trip time to at least 50 ns (7.5 m) precision for each pulse in order to catch the returning photons within the gate. More stringently, we want to exercise control over the APD gate turn-on at the 1 ns level, so that we can position the return photons appropriately in the TDC windo and within the APD gate. We can tolerate prediction offsets as long as they evolve slowly in time (e.g., at less than 1 ns per hour). This precision has been realized via use of the JPL DE403 lunar ephemeris, the latest earth-orientation and rotation information, relativistic corrections, and atmospheric correction. This document describes the procedure used in the prediction process.

2 The JPL Ephemeris

JPL has put out a number of ephemeris products, each labeled DEXXX, where XXX is a three-digit identifier. Relvant ephemerides are DE200 (year 1600 to 2169; no librations), DE403 (year 1950 to 2049; has librations), DE405 (year 1600 to 2200; has librations), DE406 (year -3000 to 3000; no nutations or librations). Each ephemeris is in essence a table of positional information for solar system bodies obtained via numerical integration of the entire solar system. The numerical integration is forced to match observations during the period that these exist, thereby establishing "initial conditions."

The actual form of the information contained in the ephemeris is not as simple as a table of positions. Rather, it is a table of Chebyshev polynomial coefficients so that for each interval in the table, one may interpolate to intermediate times to high precision. The order of the Chbyshev polynomial varies from body to body in such a way that the full precision of the numerical integration is preserved.

We use DE403, as this has the most reliable lunar libration model—according to Jim Williams. We have converted the ASCII ephemeris file spanning years 2000–2025. This gets converted to a binary file (machine dependent) that has a bit of header (identifier) information, followed by the Chebyshev coefficients in 8-byte blocks (double precision). The time intervals are broken into 32-day segments. But for many objects, the coefficients span only a sub-interval, and there are several such sub-intervals per main interval. The structure for the objects/properties of interest for us are:

Body/Property	coefficients	sub-intervals	# of components	total # of terms
Sun	11	2	3	66
Earth	13	2	3	78
Moon (geocentric)	13	8	3	312
Nutations	10	4	2	80
Librations	10	4	3	120

The moon is therefore split into 4-day sub-intervals with 13 Chebyshev coefficients per sub-interval per component (three Cartesian coordinates). Clearly the moon demands more coefficients than anyone else. In total, DE403 has 1018 coefficients per time interval, the moon and librations accounting for 42% of the data. In total, because we use the five entities in the table above, we are utilizing 64% of the ephemeris.

3 Outline of Procedure

The prediction program, given a start time, produces calculated range delays to each reflector at five-minute intervals until the moon drops below a threshold elevation (now set to 15°) or until it has run 160 points. At the end of these calculations, the data points are split into chunks no larger than 40 in size, and 8th-order polynomials fit to these segmnents. In practice, sub-nanosecond (in fact, picosecond-level) precision is achieved in the fit.

The rest of this section describes the process involved in calculating a single range-delay. The program, `moon.c`, that manages these calculations uses an iterative procedure, numbering the steps with a parameter, `n`, that runs from 0 to 14. The list below is numbered in this scheme (though offset by one, as it is impossible to get LYX to number starting at zero!). The hard part of determining position vectors of the earth and moon is accomplished by interpolating the JPL ephemeris. When the words, "get the" appear in the list below, it means: get the interpolated vector values from the ephemeris at the specified time.

1. Get the earth-moon vector from the ephemeris at the launch time, t_{L}.
2. Get the sun-earth vector from the ephemeris at t_{L}.
3. Get the sun-moon vector from the ephemeris at t_{L}. Now calculate the relativistic correction based on the vector positions established in steps 1–3
4. Get the nutations at t_{L} and apply polar correction, UT1$-$UTC correction, earth rotation transformation, nutation/precession transformation, thus establishing the station vector from the center of the earth at t_{L}.
5. Get the lunar orientaion (Euler angles) at time t_{L} and apply transformation to establish the target vector from the moon center. .
6. Get the earth position vector in the Solar System Barycenter (SSB) reference frame at t_{L}. Establish the earth station vector in the SSB frame by adding the result from step 4.
7. Get the lunar position vector in the SSB frame at t_{L}. Establish lunar target position by adding the result from step 5. Estimate upleg time from the SSB earth-station to lunar-target vector length. Add one-way relativistic and atmospheric corrections. Establish the "bounce" time, t_{B} from the relation: $t_{\mathrm{upleg}} = t_{\mathrm{B}} - t_{\mathrm{L}}$.
8. Get the lunar orienation at time t_{B} and apply transformation for updated lunar target position vector relative to the moon center.
9. Get lunar position vector in the SSB frame at time t_{B}, adding the result from step 8 to establish the lunar target SSB position vector at t_{B}. Re-calculate upleg time with relativistic and atmospheric corrections applied, establishing a revised estimate of t_{B}.
10. Update lunar orientation at revised t_{B} and update the lunar target position vector relative to the moon center.
11. Get lunar position vector in the SSB frame at revised t_{B} and add the result from step 10. Re-calculate upleg time and establish final estimate of the bounce time, t_{B}.
12. Get nutations at the estimated return time: $t_{\mathrm{R}} \approx t_{\mathrm{L}} + 2t_{\mathrm{upleg}}$.
13. Update earth pole, rotation, precession, and nutation at estimated return time $t_{\mathrm{R}} \approx t_{\mathrm{L}} + 2t_{\mathrm{upleg}}$ to establish the earth station position vector. Get earth position vector in the SSB frame at the estimated return time, t_{R}, adding the earth station vector to this. Compute the downleg time based on the final lunar target SSB vector from step 11, applying one-way relativistic and refractive corrections. Revise estimate of the return time as $t_{\mathrm{R}} = t_{\mathrm{L}} + t_{\mathrm{upleg}} + t_{\mathrm{downleg}}$.
14. Get the earth center position vector in the SSB frame at time t_{R}, adding to it an updated station vector arising from a re-computation of pole, rotation, precession, nutation transformations at the updated return time. Re-calculate t_{downleg} and update t_{R} accordingly, following the procedure in step 13.
15. As a last iteration, use the updated return time estimate, t_{R}, from step 14, repeating the steps outlined in step 14 to compute a final estimate of the downleg time. The estimated round-trip travel-time for a photon leaving the station coordinates at time t_{L} is then $t_{\mathrm{upleg}} + t_{\mathrm{downleg}}$.

4 Time

Proper handling of time is very important in carrying out predictions to nanosecond precision. Earth rotation at the latitude of the Apache Point Observatory (APO) is approximately 400 m s^{-1}. The moon, in its elliptical orbit, has a typical velocity of about 50 m s^{-1}. When the moon is low on the horizon, therefore, we may see a net range rate of about 400 m s^{-1}, translating to 0.4 mm of motion per microsecond. This means that each measurement must be referenced to absolute time to sub-microsecond precision in order for it to be useful in millimeter-level range precision. For the purpose of lunar prediction at the 1 ns level, we must know the launch time to $\sim$350 μs precision.

The JPL ephemeris uses as its time base terrestrial dynamical time (TDT), which is a continuous extension of the earlier ephemeris time (ET), but now based on atomic clocks. The international atomic time standard (TAI) is a continuous time reference whose time unit is the SI second (at mean sea level on earth) and offset from TDT by exactly 32.184 seconds (this number never changes). The sense is: one must add 32.184 seconds to TAI to get TDT. Though the ephemeris needs TDT, the normal points and the time available from the GPS clock is coordinated universal time (UTC). UTC is always an integral number of seconds away from TAI (presently 32, becoming 33 in 2006). One must add to UTC to arrive at TAI. This sense arises from the fact that the earth's rotation is slowing, so that the UTC time—which is meant to preserve the definition of noon—falls behind the steady beat of TAI. In total, one must add 64.184 seconds (soon to be 65.184) to UTC to get TDT. At present, this is hard-coded into the prediction software, but this must be changed soon to be more flexible.

In addition to handling the variable offset between UTC and TDT, one must account for the fact that TDT−UTC is always an integral number of seconds plus 0.184. This is important in determining earth orientation. Recall that in one second, the earth rotates approximately 400 m. Thus the addition of leap seconds would make the earth orientation lurch by 400 m if using UTC alone to calculate earth rotation. The time value that actually describes the earth orientation is UT1. UTC is kept within one second of UT1 via the addition of leap seconds, but UT1 can do anything it wants (or that the earth orientation wants it to do). Thus to calculate earth orientation, we make use of tabulated values of UT1−UTC (available from the International Earth Orientation Service—IERS). At present, this information (along with earth pole and nutation-correction information) is obtained from the IERS via a script called `autoget`, which places results in the file `eopc04.0X`, where `X` is the year number.

When computing a predicted round-trip range on the fly, the time is established as follows:

1. The laser fire is accompanied by a fiducial gate event, the closing of which latches the time-within-second (TWS) counter.
2. After a new second, the first gate event to close laches the GPS clock to the nearest microsecond
3. The last-obtained GPS time (rounded to the nearest second) is added to the TWS count for a given laser fire to establish the fire time to 20 ns resolution.
4. This fire time (in UTC) is converted to day of year plus fractional day.
5. The polynomial fit to the range calculations comes with a start time, in day of year plus fractional day format. This value is subtracted from the fire time (in the same units) to establish a time variable in fractional days for use in the polynomial.

Meanwhile, the times used in the generation of calculated ranges are UTC times. Each time the ephemeris is called, the TDT−UTC offset is applied to the time variable in the function call.

5 Earth Orientation

The JPL ephemeris provides information on center-to-center vectors within the solar system, but leaves earth orientation alone, aside from providing nutation values. This is because the earth orientation is not entirely deterministic. Angular momentum exchange between atmosphere and oceans, in addition to mass redistribution of the crust and mantle result in an erratic, unpredictable:

- motion of the rotation axis with respect to the earth body
- rotation rate of the earth

- tipping of the rotation axis with respect to the inertial frame (in addition to the deterministic precession/nutation)

These vagaries are measured and tracked by the IERS, and are referred to as earth orientation parameters (EOPs). The latest values of these parameters are available from the FTP site: `hpiers.obspm.fr:eop-pc/eop/eopc04/` with a filename of `eopc04.XX`, where `XX` is the two-digit year number (e.g., `05`). Typical values for the polar offsets (in x and y) are a few tenths of an arcsecond. Typical integrated UT1−UTC offsets (as discussed above, and which relate to the integrated rotation angle) are less than one second, and typical nutation corrections are several hundredths of an arcsecond. Each of these parameters evolves slowly, over timescales of about a year.

5.1 Prescription for Orientation

There is a specific sequence that one follows to rotate an earth station vector into the solar-system barycentric frame:

1. Rotate the station vector by the polar offset to establish corrected geocentric coordinates prior to rotating about the shifted axis.
2. Rotate the earth (station vector) about the true axis through an angle based on UT1.
3. Apply precession and nutation rotations to the earth axis, including the corrections to nutation.

The first step is a straightforward matrix rotation about the earth's x and y axes by very small angles. The second step is a single rotation about the earth's z axis by an angle proportional to UT1. The third step follows a complex prescription established by the IERS.

6 Lunar Orientation

The lunar orientation is provided in the JPL ephemeris as a set of three Euler angles. Two of these angles in essence describe the polar orientation, and the third describes the rotation angle of the moon. This last angle becomes large, incrementing by 2π every month. The effect of librations are implicitly embedded in these Euler angles, though it should be noted that librations come in two flavors. Optical librations are merely consequences of vantage point as the moon travesl fast then slow in its elliptical (and inclined) orbit, so that we on earth *see* slightly different faces of the moon from our varying vantage point. These librations account for almost all of the total libration, with typical amplitudes of 7°. Physical librations, on the other hand, result from torques on the lunar body that physically re-orient the lunar pole (and rotation) in space. These are much smaller in size, never amounting to more than about 0.04°. Only the physical librations are represented in the JPL ephemeris. The optical librations are automatically "generated" when the earth-moon vector and lunar orientation vector are simultaneously considered.

7 Relativistic Correction

Time runs more slowly the deeper one is in a gravitational potential. The JPL ephemeris is based on a coordinate time in the solar system barycentric frame, while the time we measure is referenced to the atomic clock interval (the SI second) as it runs on the earth at mean sea level. A clock ticking off SI seconds in the center of the solar system runs more slowly than our clock on earth. Thus our earth clock ticks off more nanoseconds when measuring a round-trip travel time than would be indicated by the raw ephemeris result. If a light signal is emitted from x_1 at time t_1, and is then received at position x_2 at time t_2, the corrected propagation time is:

$$t_2 - t_1 = \frac{|\vec{x}_2(t_2) - \vec{x}_1(t_1)|}{c} + \sum_j \frac{2GM_j}{c^3} \ln\left(\frac{r_{j1} + r_{j2} + \rho}{r_{j1} + r_{j2} - \rho}\right),$$

where j is the body number, $\vec{x}_j$ is the body center, $r_{jk} = |\vec{x}_j - \vec{x}_k|$, and $\rho = |\vec{x}_2 - \vec{x}_1|$. The contribution from the sun dominates, with typical one-way corrections around 25 ns, varying by ± 1.5 ns depending on the lunar true anomaly (i.e., distance). The earth potential contributes about 0.12 ns, and the moon's potential contributes about 2 ps.

To understand the nature of this time correction a little more, pretend we're at quarter moon, so that $r_{s1} = r_{s2} = r$, where the subscript s denotes the sun. The fraction in the logarithm is then $\frac{2r+\rho}{2r-\rho}$, which reduces to

$(1+\frac{\rho}{2r})(1+\frac{\rho}{2r}) \approx 1+\frac{\rho}{r}$ if ρ is small compared to r. Then the logarithm is just ρ/r. Now the solar correction term becomes $2(GM_s/rc^2)(\rho/c)$,which is just to say that the light travel time between $\vec{x}_1$ and $\vec{x}_2$ ($\tau = \rho/c$) is modified by $2GM_s/rc^2$. This dimensionless quantity is familiar in general relativity a describing the strength of the potential that modifies Minkowski flat spacetime. This ratio for the sun's potential at the distance of the earth amounts to 2×10^{-8}, modifying the 1.25 second one-way travel time by 25 ns.