LLR analysis with INPOP

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History

- INPOP05:
  - designed to be similar to DE405
  - same model and parameters
  - goal: validate the dynamical model

- INPOP06:
  - dynamical model improved
  - fitted to planetary observations
  - Lunar motion constrained by DE405

- INPOP08a:
  - dynamical model improved
  - more planetary observations (+fitting method)
  - fitted to LLR observations

- INPOP10a:
  - more planetary observations (+fitting method)
  - More LLR observations

Dynamical model

State vector contains:

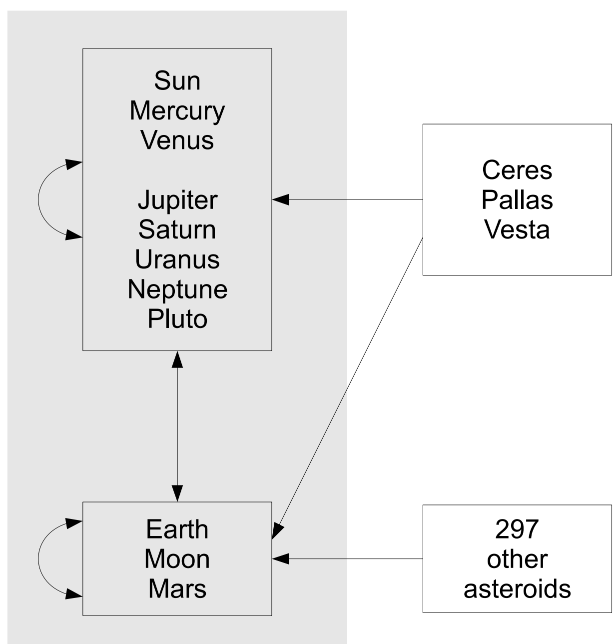
- Solar System barycentric positions/velocities of Sun, planets, Pluto
- Solar System barycentric positions/velocities of 300(+) asteroids
- Geocentric positions/velocities of the Moon
- Euler's angles of the Moon
- *orientation of the Earth (I06 → …)*
- *asteroid ring (I06 → …)*
- *TT-TDB transformation (I08 → …)*

Numerical integration:

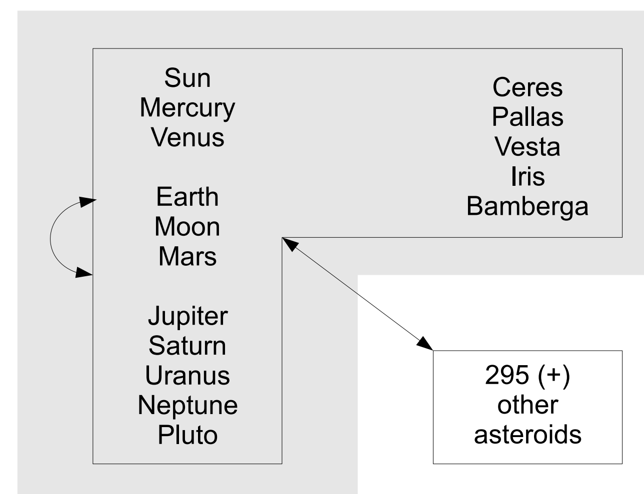
- Adams method (order 12)
- initialisation with ODEX
- extended precision on IA64 (80b)
- fixed step size (~0.055 day)

Dynamical model: point-mass interactions

DE405



INPOP06 → ...



Newtonian forces: →, ←, ↔

Relativistic corrections:

Dynamical model:
figure interactions $\leftrightarrow$ point-mass

$$U(r,\varphi,\lambda) = -\frac{GM}{r}\sum_{n=0}^{+\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n P_{nm}(\sin\varphi)\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right)$$

Time varying coefficients due to tides, spin, post-glacial rebound

→ forces: acceleration of extended and pertubating bodies
→ torques: angular momentum of extended body

- Sun (J_2) ↔ planets, Moon (forces only)
- Earth (J_2,J_3,J_4) ↔ Sun, Moon, Venus, Jupiter (+*other planets for torques*)
- Moon $(C,S)_{2m,3m,4m}$ ↔ Earth, Sun, Venus, Jupiter (forces and torques)

Dynamical model: solid tides effects

$$\Delta U = -\frac{GM}{r}\sum_{n=2}^{+\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}P_{nm}(\sin\varphi)\left(\Delta C_{nm}\cos m\lambda + \Delta S_{nm}\sin m\lambda\right)$$

$$\begin{cases}
\Delta C_{20} = \dfrac{m_g}{M}\left(\dfrac{R}{r_g^*}\right)^3\dfrac{k_{20}}{2}\dfrac{2r_z^{*2} - r_x^{*2} - r_y^{*2}}{r_g^{*2}}\\
\Delta C_{21} = \dfrac{m_g}{M}\left(\dfrac{R}{r_g^*}\right)^3 k_{21}\dfrac{r_x^* r_z^*}{r_g^{*2}}\\
\Delta C_{22} = \dfrac{m_g}{M}\left(\dfrac{R}{r_g^*}\right)^3\dfrac{k_{22}}{4}\dfrac{r_x^{*2} - r_y^{*2}}{r_g^{*2}}\\
\Delta S_{21} = \dfrac{m_g}{M}\left(\dfrac{R}{r_g^*}\right)^3 k_{21}\dfrac{r_y^* r_z^*}{r_g^{*2}}\\
\Delta S_{22} = \dfrac{m_g}{M}\left(\dfrac{R}{r_g^*}\right)^3\dfrac{k_{22}}{2}\dfrac{r_x^* r_y^*}{r_g^{*2}}
\end{cases}$$

tide generating body delayed coordinates:

$$\vec{r^*} = {}^t(r_x^*, r_y^*, r_z^*) = \vec{r}(t - \tau_{nm})$$

Earth (Moon*, Sun*) ↔ Sun, Moon, Venus, Jupiter (+ *other planets for torques*)

← k_{20}, k_{21}, k_{22}, τ_{20}, τ_{21}, τ_{22}

Moon (Earth*, Sun*) ↔ Earth, Sun, Venus, Jupiter (forces and torques)

← k_M, τ_M

Dynamical model: spin deformation

$$\Delta U = -\frac{GM}{r}\sum_{n=2}^{+\infty}\left(\frac{R}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\varphi)\left(\Delta C_{nm}\cos m\lambda + \Delta S_{nm}\sin m\lambda\right)$$

$$\begin{cases} \Delta C_{20} = \dfrac{k_{20}R^3}{3GM}\dfrac{1}{2}\left(\omega^{*2} + \overline{\omega}^2 - 3\omega_z^{*2}\right) \\ \Delta C_{21} = -\dfrac{k_{21}R^3}{3GM}\omega_x^*\omega_z^* \\ \Delta S_{21} = -\dfrac{k_{21}R^3}{3GM}\omega_y^*\omega_z^* \\ \Delta C_{22} = \dfrac{k_{22}R^3}{3GM}\dfrac{1}{4}\left(\omega_y^{*2} - \omega_x^{*2}\right) \\ \Delta S_{22} = -\dfrac{k_{22}R^3}{3GM}\dfrac{1}{2}\omega_x^*\omega_y^* \end{cases}$$

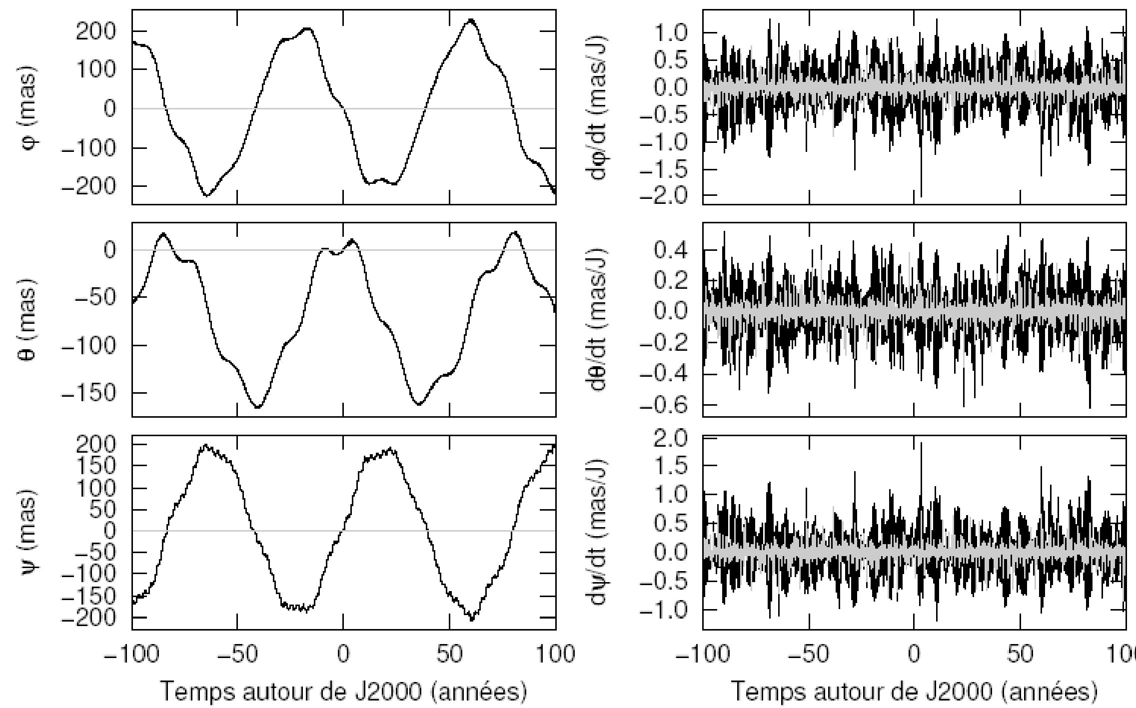
Delayed instant vector of rotation

Dynamical model: figure-figure effects

Standish (ssd.jpl.nasa.gov/pub/eph/planets/ioms/ExplSupplChap8.pdf):

$$\overrightarrow{M}_{fig-fig} = \frac{15\mu_e R_e^2 J_{2e}}{2r_e^5}\left\{\left(1-7\sin^2\phi\right)\vec{r}_e\wedge I\vec{r}_e + 2\sin\phi\left(\vec{r}_e\wedge I\vec{P}_e + \vec{P}_e\wedge I\vec{r}_e\right) - \frac{2}{5}\vec{P}_e\wedge I\vec{P}_e\right\}$$

Torque exerted by the Earth on the Moon
Force is neglected



Main differences with DE405: orientation of the Earth

DE405: kinematic forcing (precession – nutation model)

INPOP (I06 → …)
Modelized by its angular momentum:

$$\dot{\vec{G}} = \vec{M}_2 + \vec{M}_3 + \vec{M}_4 + \vec{M}_{tides} + \vec{M}_{GP}$$

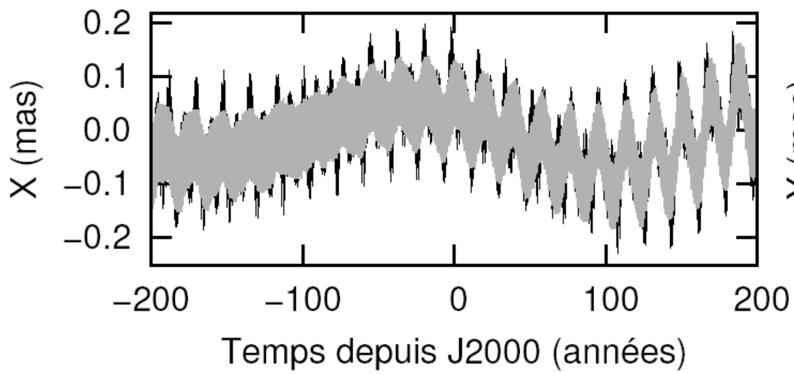
•torques due to
- figure ↔ point-mass (including J2 dot)
- tides
- geodesic precession

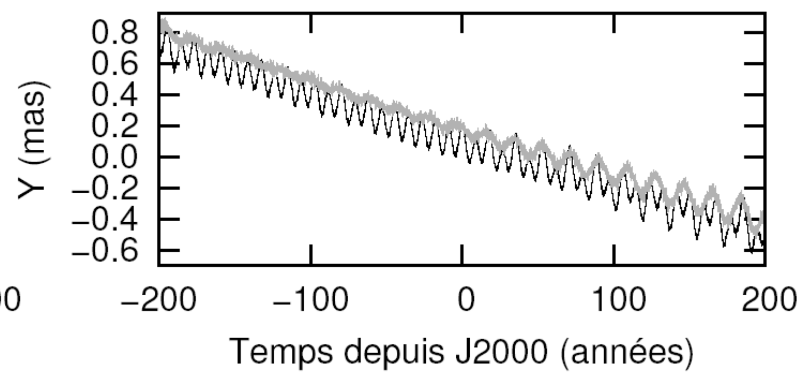
•integrated together with equations of motions of bodies
•initial conditions and C/MR2 ratio fitted to REN2000-P03 (200 years around J2000)

REN2000: rigid Earth nutations of Souchay et al. (1999)
P03: precession of Capitaine et al. (2003)

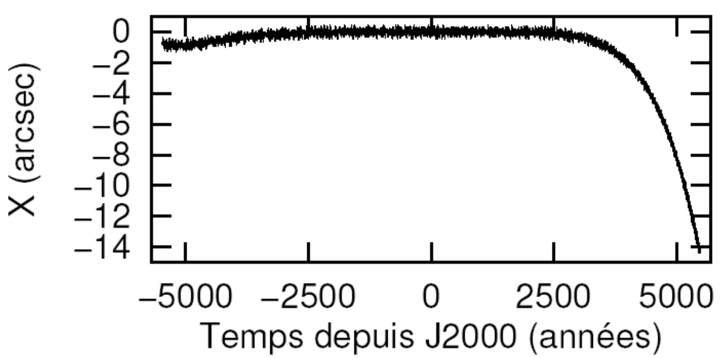
Main differences with DE405: orientation of the Earth

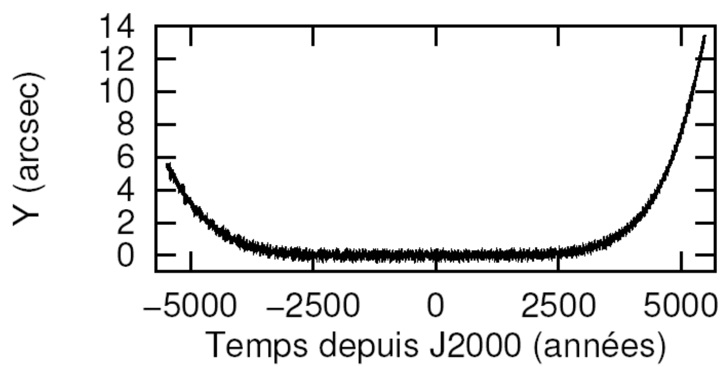
Differences between INPOP's integration and REN2000-P03
X and Y are the Earth's pole coordinates in ICRF.





Differences between INPOP's integration and CIP-P03
X and Y are the Earth's pole coordinates in ICRF.





Main differences with DE405: asteroid ring

- DE405: none
- but DE414 → ...
  - Fixed to Solar System Barycenter ?
  - Equations ? Krasinsky (2002) ?

- INPOP06:
  - fixed to Solar System Barycenter
  - Krasinsky (2002) extended to outer planets
  - not isolated system → problem on long term solutions

- INPOP08 → … :
  - Kuchynka et al., (2010) (inclinaison)
  - moves with the center of the Sun
  - orientation integrated
  - forces and reactions with planets and Moon
  - → isolated system, small drift of Solar System barycenter
  - → allows long term integrations

Main differences with DE405: TT-TDB transformation

- no effect on motion but usefull for data reduction
- solution of a « differential equation »
- depends on positions, velocities, accelerations and « masses » of bodies
- very convenient to integrate together with equations of motions

Klioner et al., (2010):

$$\frac{d\,(TT - TDB)}{dTDB} = \frac{1 - L_G}{1 - L_B}\left(1 + \frac{\alpha}{c^2} + \frac{\delta}{c^4}\right) - 1$$

$$\alpha = -\frac{1}{2}v_T^2 - \sum_{A \neq T} \frac{\mu_A}{r_{TA}}$$

sum on all bodies except the Earth

$$\delta = -\frac{1}{8}v_T^4 + \left(\beta - \frac{1}{2}\right)\left(\sum_{A \neq T}\frac{\mu_A}{r_{TA}}\right)^2$$

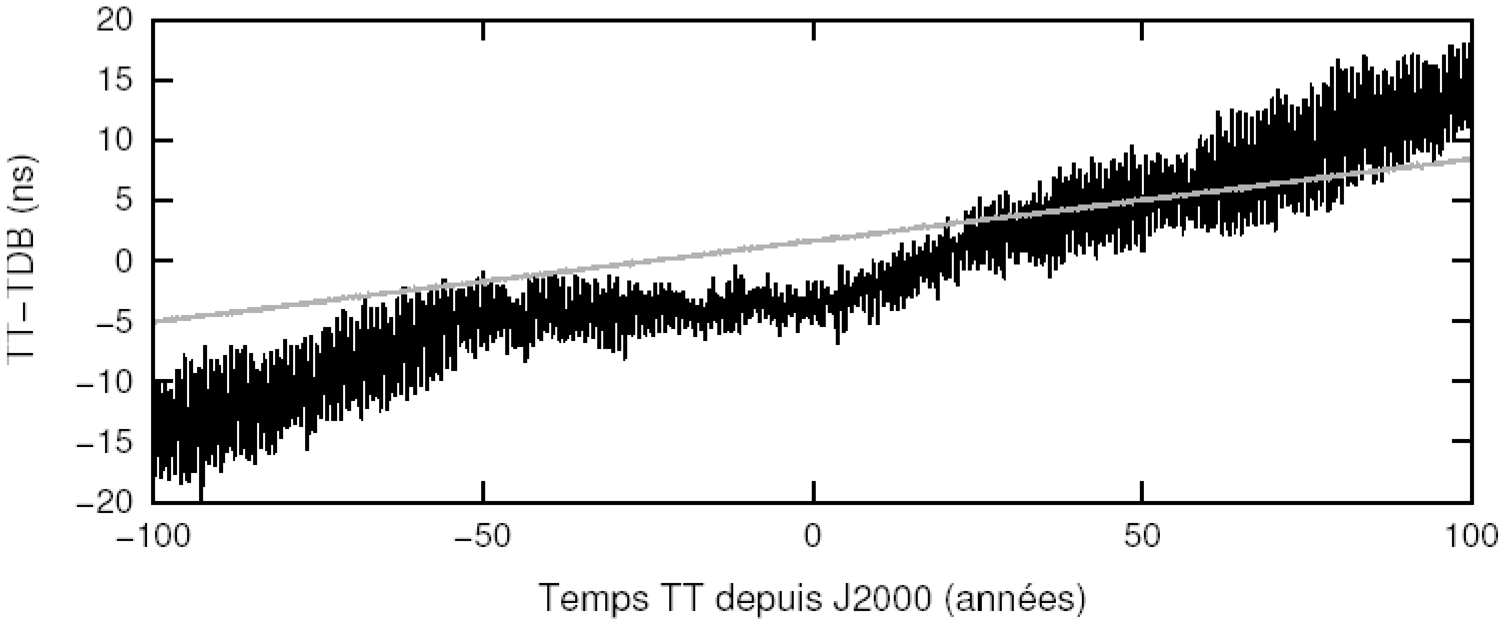
sum on all « relativistic bodies »

$$+ (2\beta - 1)\sum_{A \neq T}\left(\frac{\mu_A}{r_{TA}}\sum_{B \neq A}\frac{\mu_B}{r_{BA}}\right)$$

acceleration due to all effects

$$+ \sum_{A \neq T}\frac{\mu_A}{r_{AT}}\left[2\,(1+\gamma)\,\vec{v}_A \cdot \vec{v}_T - \left(\gamma + \frac{1}{2}\right)v_T^2 - (1+\gamma)\,v_A^2 + \frac{1}{2}\ddot{\vec{r}}_A \cdot \vec{r}_{TA} + \frac{1}{2}\left(\frac{\vec{v}_A \cdot \vec{r}_{AT}}{r_{AT}}\right)^2\right]$$

Main differences with DE405: TT-TDB transformation

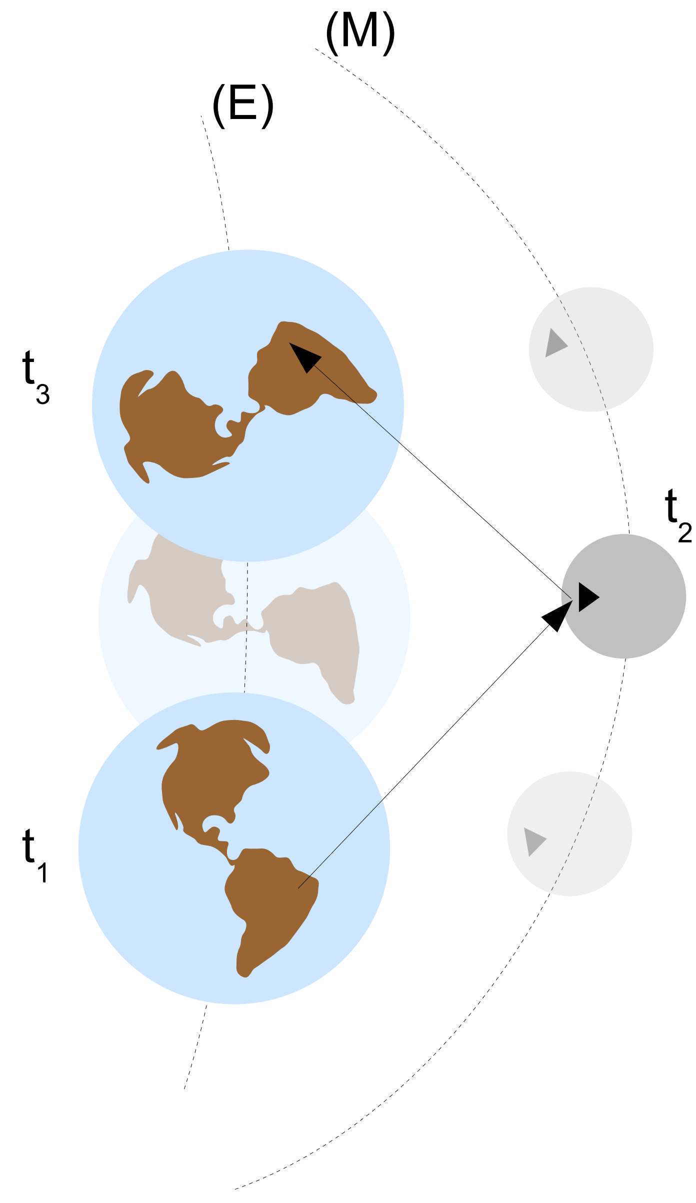


Differences on TT-TDB (ns):
grey: TE405 (Irwin & Fukushima 1998) - INPOP08
black: SOFA (Fairhead & Bretagnon 1990 corrected) - INPOP08

Small differences, no significant effect on residuals
But consistency between timescale and motions (4D ephemeris)

LLR reduction model

$$\Delta T_a = \frac{\|\overrightarrow{BM}_2 + \overrightarrow{M_2 R}_2 - (\overrightarrow{BE}_1 + \overrightarrow{E_1 S}_1)\|}{c} + T^{RG} + T^{atm}$$



B: Solar System barycenter
M: center of mass of the Moon
E: center of mass of the Earth
S: station
R: reflector

t_1: emission
t_2: reflection
t_3: reception

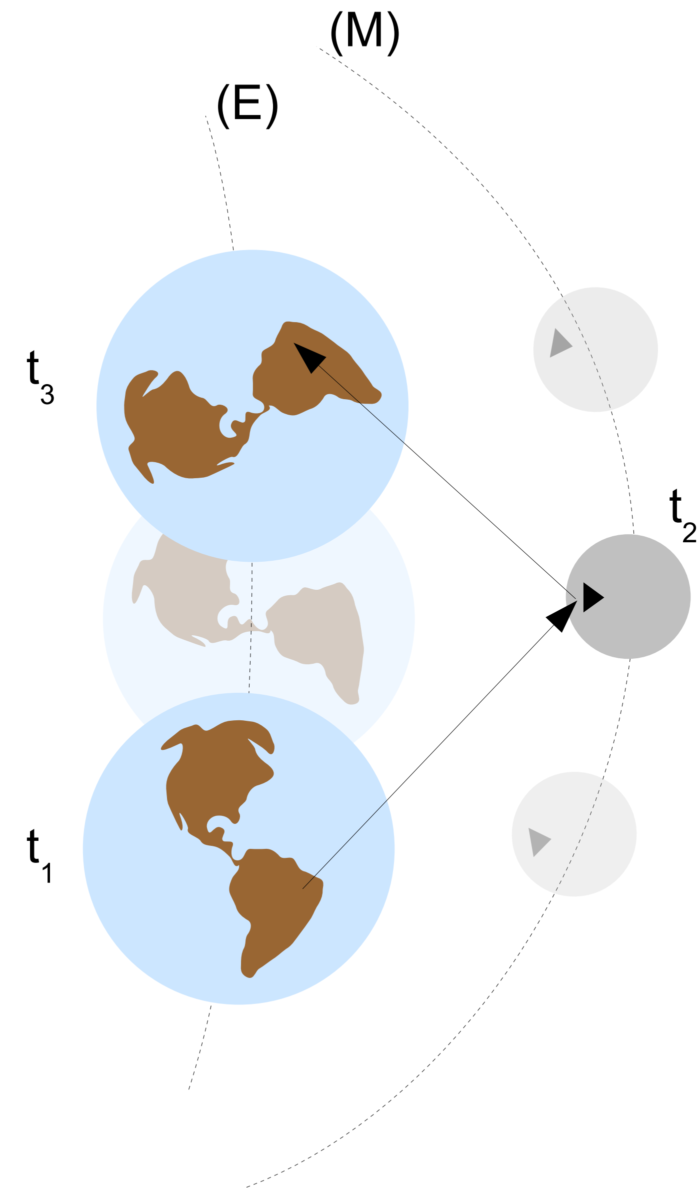
$t_2 = t_1 + \Delta T_a$ → implicit equation → iterations

Same method for downleg time (1 → 3)

LLR reduction model

$$\Delta T_a = \frac{\|\overrightarrow{BM}_2+\overrightarrow{M_2R}_2-(\overrightarrow{BE}_1+\overrightarrow{E_1S}_1)\|}{c}+T^{RG}+T^{atm}$$

INPOP(t_2) INPOP(t_1)



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Station's geocentric position (IERS Conventions 2003):

•ITRF2005 coordinates (from ITRF/IGN website, but some are not available !)

•Displacement due to

- tectonic plate motion (from ITRF/IGN website)
- tides effects (V. Dehant's subroutine)
- ocean loading (D. Agnew's subroutine)
- atmospheric loading (IERS Conventions 1996)
- polar tide (IERS Conventions 2010)

•Transformation GTRF → GCRF (IERS 2003: CIP + EOP of the C04 serie)

•Transformation GCRF to BCRF (Lorrentz contraction)

LLR reduction model

$$\Delta T_a = \frac{\|\overrightarrow{BM_2} + \overrightarrow{M_2 R_2} - (\overrightarrow{BE_1} + \overrightarrow{E_1 S_1})\|}{c} + T^{RG} + T^{atm}$$

Reflector's selenocentric position:

- coordinates in principle axis reference frame
- displacement due to tides effects
- displacement due to spin
- transformation principle axis reference frame → « MCRF » using INPOP(t_2) librations
- transformation « MCRF » to BCRF (Lorrentz contraction)

LLR reduction model

$$\Delta T_a = \frac{\| \overrightarrow{BM_2} + \overrightarrow{M_2 R_2} - (\overrightarrow{BE_1} + \overrightarrow{E_1 S_1}) \|}{c} + T^{RG} + T^{atm}$$

Shapiro's delay (relativistic light deviation) - Williams (1996):

$$T^{RG} = \frac{1+\gamma}{c^3}\mu_S \ln\left(\frac{r_1^S + r_2^S + r_{12}^S + (1+\gamma)\frac{\mu_S}{c^2}}{r_1^S + r_2^S - r_{12}^S + (1+\gamma)\frac{\mu_S}{c^2}}\right) + \frac{1+\gamma}{c^3}\mu_T \ln\left(\frac{r_1^T + r_2^T + r_{12}^T}{r_1^T + r_2^T - r_{12}^T}\right)$$

Only Solar's and Earth's contributions

LLR reduction model

$$\Delta T_a = \frac{\| \overrightarrow{BM_2} + \overrightarrow{M_2 R_2} - (\overrightarrow{BE_1} + \overrightarrow{E_1 S_1}) \|}{c} + T^{RG} + T^{atm}$$

Time delay due to troposphere (Marini & Murray 1972):

- true elevation
- atmospheric conditions (temperature, pression, humidity)
- laser wavelength
- position of the station

Mendes and Pavlis (2004) tested by S. Bouquillon (SYRTE)

LLR fit to observations

Parameters involved in LLR measurements (188)

- positions of reflectors
- positions and velocities of stations
- Moon's initial conditions (position, velocity and librations)
- EMB's initial conditions (position and velocity)
- Stokes coefficients (up to 4^{th} degree)
- time delays, Love numbers (Earth, Moon)
- post-newtonian parameters
- offsets applied on some observations (40x2)

But some of them are:

- not independent (transmission and reception stations of Haleakala)
- better determined with planetary observations (M_E/M_M, EMB's initial conditions)
- better determined with an another technique (VLBI → motion of stations)
- are badly determined: $S_{43} = (-2.0 \pm 13.5) \times 10^{-6}$

Selection of fitted parameters

Iterations with elimination of the parameter having the greatest ratio error/value
→ increase of residuals (but weak)
→ decrease of formal errors on other parameters

Solution:		S074	…	S065	…	S059	…	S055	…	S051
Maximum ratio		750%	…	9%	…	3.6%	…	1.2%	…	0.3%
Station	Period	σ (cm)	…	σ (cm)	…	σ (cm)	…	σ (cm)	…	σ (cm)
Grasse (1)	1984-1986	15,9	…	15,9	…	16,0	…	15,6	…	16,2
Grasse (2)	1987-1995	6,3	…	6,3	…	6,4	…	6,0	…	8,2
Grasse (3)	1995-2010	3,7	…	3,7	…	4,0	…	5,4	…	6,9
Mc Donald	1969-1985	31,2	…	31,4	…	31,8	…	36,1	…	50,0
MLRS1 (1)	1982-1985	73,3	…	73,0	…	73,3	…	72,5	…	71,7
MLRS1 (2)	1986-1988	8,0	…	7,5	…	7,3	…	7,4	…	9,8
MLRS2 (1)	1988-1999	4,3	…	4,3	…	4,3	…	4,3	…	6,5
MLRS2 (2)	1999-2008	4,6	…	4,6	…	4,8	…	4,9	…	6,5
Haleakala	1984-1992	8,1	…	8,2	…	8,1	…	8,4	…	11,6
Apollo	2006-2009	4,8	…	4,9	…	4,9	…	5,3	…	7,1

formal error (1-σ) on C_{33M} : 6.8×10^{-7} → 3.3×10^{-8} → 6.3×10^{-9} → 5.2×10^{-9} → 4.6×10^{-9}

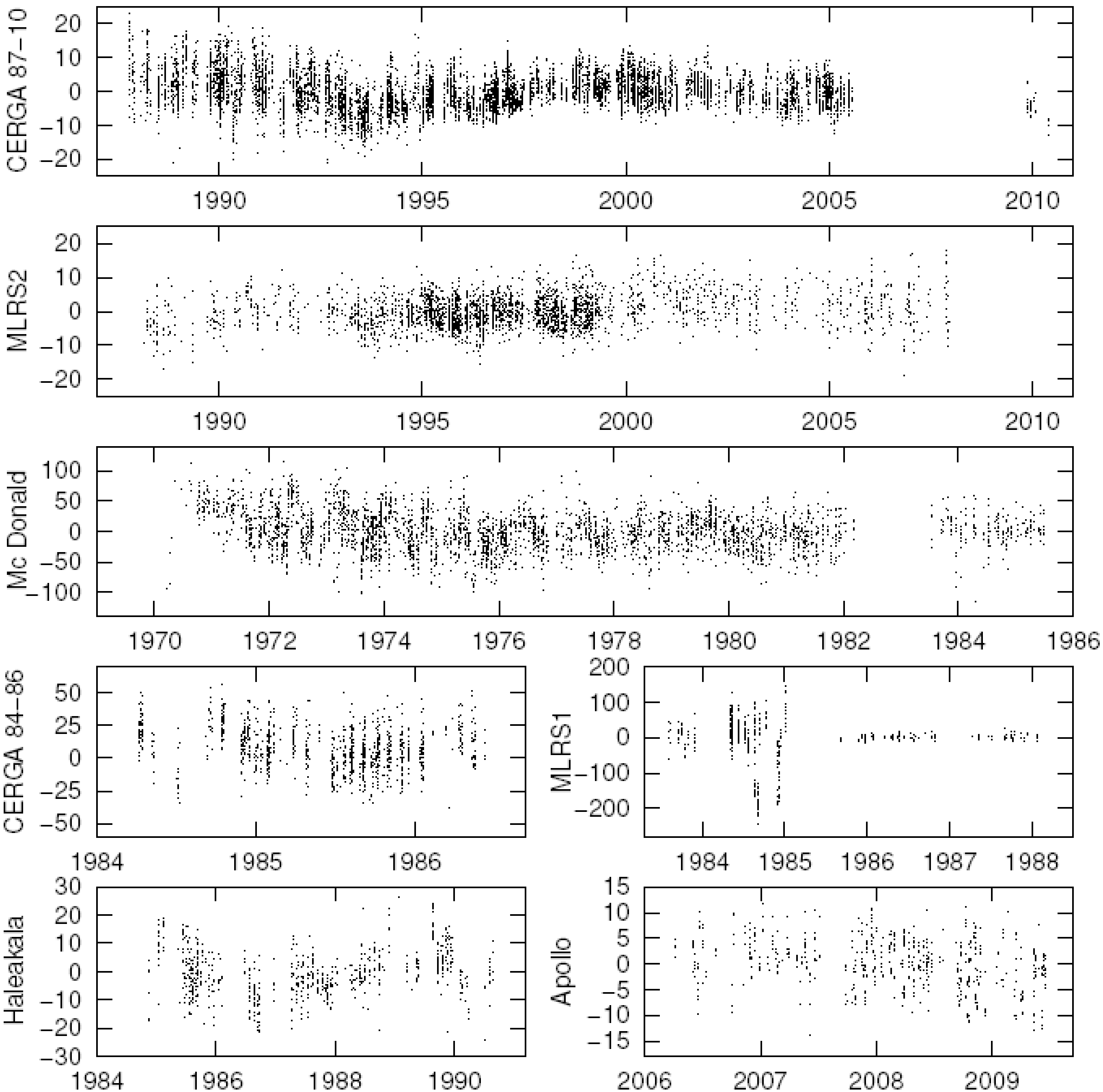
Choice: maximum ratio <5% → 59 parameters fitted

Fitted parameters

- Earth-Moon vector at J2000 (6)
- Moon's libration at J2000 (6)
- Reflectors coordinates (3x4=12)
- Stations coordinates (6x3=18)
- Earth's time delays τ_{21} and τ_{22} (2)
- Moon's Love number k_2 and time delay τ_M (2)
- Earth's potential coefficients J_2 and J_3 (2)
- Moon's potential coefficients
  - C_{20} and C_{22} (2)
  - C_{3m} and S_{3m} except C_{32}, S_{33} (5)
  - C/MR^2 (1)
- GM_{EMB} (1)
- Biaises (2)

Others:
Ratio > 5% between formal error and fitted value

INPOP10a LLR residuals



Problem:
strong signal on CERGA and Haleakala

Dynamical model ?
same with DE418 → DE423

Reduction process ?
same as SYRTE

Residuals comparison INPOP10a / DE423

		INPOP10a	*DE423*
Station	Period	σ (cm)	σ (cm)
CERGA (1)	1984-1986	16,0	*14,7*
CERGA (2)	1987-1995	6,4	*5,9*
CERGA (3)	1995-2010	4,0	*3,9*
Mc Donald	1969-1985	31,8	*29,8*
MLRS1 (1)	1982-1985	73,3	*70,3*
MLRS1 (2)	1986-1988	7,3	*6,1*
MLRS2 (1)	1988-1999	4,3	*4,7*
MLRS2 (2)	1999-2008	4,8	*4,6*
Haleakala	1984-1992	8,1	*8,1*
Apollo	2006-2009	4,9	*4,7*

DE423:

planetary and lunar motion fixed
fit of parameters only involved in the reduction of observations
data reduction with IMCCE's procedures (JPL's reduction even better)

DE423 residuals better than INPOP10a ← lunar core ?

LLR Perspectives

Validate the reduction model
→ understand the signal on CERGA's data

Constraints by new LLR observations
→ Lunokhod 1 !!!

Constraints by other data type
→ Lunar Prospector ?
→ Kaguya ?

Improve the dynamical model
→ lunar core ?
→ lense thirring effect ?

Tests of gravitation model
→ done for years by A. Fienga with planetary observations
→ just began with LLR data