Renewable Energy Estimates

In Lecture 2, we looked at the scale of current energy use, and the total amount of solar energy hitting the surface of the Earth. We found that we receive about 10,000 times as much sunlight as the 15 TW global scale of energy "production." Now we will use order-of-magnitude techniques to estimate how much we might get from alternative sources. As a number for reference, 15 TW divided by the surface area of the Earth is 0.03 W/m^2, or about 0.1 W/m^2 over land alone.

Disclaimer: The estimates that follow are not to be taken as definitive numbers that you can take to the bank. They are made in the spirit of the course—from guesswork and things already known. I may have occasionally consulted external sources *after* the fact to make sure I was not wildly off. More often than not, such checks result in smiles, not grimaces.

Solar Power

Direct sun has 1000 W/m^2 reaching the ground. Averaged over day, night, season, and weather, this is reduced to about 200 W/m^2 (factor of 4 in geometry alone, without weather).

Solar Photovoltaic

PV converts photons into photo-electrons (current) directly in the silicon. With a band-gap of 1.1 eV, light longward of 1.1 μm ($\lambda = hc/E$) is ineffective. A crude sketch of the Planck function peaking in the visible band can easily convince you that 25% of the light is lost to the infrared portion. A photon blue-ward of 1.1 μm will have excess energy that gives the liberated electron excess kinetic energy (heat). So at the peak of the blackbody curve for the sun, the photon energy is twice what is needed, and only half will contribute to the photo-current. Thus from a purely physical basis, one might expect PV panels to do no better than 35% efficiency. Recombination and other maladies chop this further, so that affordable panels today come in around 16%.

Considering land only (about a quarter of the globe), and using PV panels at 16% efficiency, we could reap 4% of the 10,000 $\times$ excess if outfitting all our land, so we only need $1/400^{\text{th}}$, or 0.25% of land covered to satisfy our current needs (all forms of energy, including transportation and industry). To address possible confusion, the fact that a given solar panel is not always in the sun is already accounted for in the factor of 10,000 number, which builds in a factor-of-four for surface area vs. projected area of the Earth. Another approach says a 15% efficient panel getting a day/night/season/weather average of 200 W m^{-2} effectively produces 30 W m^{-2}. Times 0.25% of land area (about 3×10^{11} m^2) gives something on the order of 10 TW, which is commensurate with global energy demand.

How much rooftop do we have? If everyone in the U.S. has at least 10 m^2 over their heads, including work space (about one bedroom; keep in mind urban high-rises driving the average down), then we have 3×10^9 m^2 of roof-space. The U.S. is approximately 4000 km by 2500 km, or 10^{13} m^2. $1/400^{\text{th}}$ of this is 2.5×10^{10} m^2, or a factor of ten larger than roof space. How much pavement do we have? Go figure...

Solar Thermal

Solar thermal power plants use steered mirrors or parabolic troughs to concentrate sunlight and heat a fluid that will then heat water to drive a steam turbine. The heat engine itself has a Carnot efficiency in the neighborhood of 40%, and then there are engineering losses on top of this, so that we might estimate a total efficiency of 20% to get electricity: not much different from solar PV. But it's a little worse because you *must* steer the mirrors or troughs to put the concentration where it needs to go, meaning the articulated mirrors cannot shadow-block each other, and therefore must allow substantial "elbow" room. So the land area is not as efficient as PV, though the technique is economically competitive with other renewables when land is cheap (and sun-scorched: solar thermal needs direct sun and cannot tolerate diffuse, cloudy light the way PV can). Still, the fraction of land needed is well shy of unity, and solar thermal lends itself well to built-in thermal storage.

Wind

Wind is a secondary source: the crumbs of solar input due to differential heating. So the total budget must be some small fraction of the total sunlight available. We might guess 1% in a hasty estimation. Or if we say that the wind is driven by convective flow, its maximum thermodynamic efficiency would be $(T_\mathrm{h} - T_\mathrm{c})/T_\mathrm{h} \approx 50/300 \sim 0.15$. If half of the atmospheric cooling is convection and half radiation, and assuming some viscous loss, we're looking at something in the ballpark of 5%. But let's come at it from a different direction.

Weather/wind will be found in the troposphere, about 12 km thick (my guess: about 40 kft). At altitude, we might guess an average wind speed to be 20 m/s (44 m.p.h.). Maybe this is high (certainly is high for the average wind speed at ground level). Each square meter of land has about 10^4 kg of air sitting above it (atmospheric pressure is 10^5 Pa, at $g = 10$ m/s^2). So each square meter has $\frac{1}{2}mv^2 \approx 2 \times 10^6$ J of kinetic energy over it at any given time. But if we sapped this energy out of the atmosphere in one second—extracting 2 MW for each square meter over the globe—it would take a good while to re-establish. Let's guess a day, or 8×10^4 seconds. Thus we might estimate the power in wind to be 25 W m^{-2}. This estimate seems a little high, at about 10% of average solar input, but let's roll with it.

We can't pull all the power out of wind, because to do so would be to constantly stall its generation. Let's then say we can get half of it (still greedy?). And we also can't practically extract wind to towering heights. Let's say we're restricted to the lower 150 m. So we get $1.3\,\mathrm{kg/m^3} \times 150\,\mathrm{m^3} = 200\,\mathrm{kg}$ of the atmosphere per square meter, or 2% of the total. Combining with our 50% above, we get 1% of the 25 W/m^2, or 0.25 W/m^2. If we consider that the average wind speed at the ground is well short of the high-altitude average, and that the energy scales as v^2, we may even reduce this to 0.1 W/m^2 or lower—about 0.05% the average sunlight delivered to the ground.

If the gross solar input is 10,000 times bigger than our use, then wind power is about 4 times our use, or about equal to our use if we just consider the wind over land. Why would I say this is optimistic? Because the initial estimate had 10% of the solar energy making wind in the atmosphere, which seems too far off target. We reduced our numbers later to account for slower speed near the ground, but this reduction would be required in any case.

See the reference by de Castro et al. for a very nice order-of-magnitude estimate (more sophisticated than the one here) that also considers many practical aspects, ending up with 1 TW as the realizable global wind potential.

Geothermal

Decay of thorium, uranium, and potassium in the Earth provide a source of heat flow to space. We know that surface caves have a stable temperature around 15°C, and that the mantle must be a rock-melting temperature of at least 1000 C about 40 km down (crust thickness). So the gradient would be about 25 C/km. Rock has a thermal conductivity somewhere between wood, at ~ 1 W/K/m and metal at ~ 100 W/K/m, and we'll guess 5 W/K/m to be somewhat closer to wood than heat-sucking metal. This means each square meter passes $(0.025\,\mathrm{K/m}) \cdot (5\,\mathrm{W/K/m}) \approx 0.125\,\mathrm{W/m^2}$ of heat. [After-note: using more realistic thermal conductivities of rock results in about half this flow rate.]

This is comparable to the land-area power density of our global society. But it is a diffuse source without much ΔT to offer except in rare places of heightened geothermal activity. To run a heat-engine, the efficiency goes like $\Delta T/T_\mathrm{hot}$. Even if we span a kilometer to get 25 K, the theoretical maximum thermodynamic efficiency is less than 10%. Not much of a starter given the diffuse, weak nature—except in a few isolated areas.

Tidal

Tidal flows can be trapped and made to flow through turbines as the height of the tide creates a pressure head. In mid-ocean, peak-to-peak tides are less than one meter. Continental shelves amplify this due to sloshing and resonant behavior. A few exceptional places might see amplification to levels in excess of 10 m.

To put a scale on the whole phenomenon, imagine a quarter of the planet's surface is raised one meter on both sides of the prolate bulge. This would be about 10^{14} m^2, with an average height increase of 0.5 m for a total gravitational potential energy of 0.5×10^{18} J. If we could trap and release this twice daily (4×10^4 s), we would derive about 10 TW of power—just shy of our global appetite. But this is unattainable from a practical standpoint.

Another scale is to imagine trapping the water on continental shelves within 50 km of land, where the height may more typically be 2 m (average height then 1 m). Not all coastlines have continental shelves, but we might imagine that such shorelines would be enough to circle the globe once, or about 40,000 km. Now the potential energy is 4×10^{16} J, delivered every 4×10^4 s for a power of 1 TW. And if you thought the Great Wall of China was impressive!

A final scale we might seek comes from the current rate of tidal dissipation. A little bird told me that the Moon recedes 3.8 cm per year in its orbit. The change in potential is then $GMm\Delta r/r^2$, and the kinetic energy changes by half this (virial thm.), so the total energy change is half this amount. It is handy to know that $GM_\oplus = 3.98 \times 10^{14} \approx 4 \times 10^{14}$ in SI units, that the Moon is $1/80^{\text{th}}$ the mass of the Earth (Earth's mass is 6×10^{24} kg), and that the Moon is 1.25 light-seconds away, or about 4×10^8 m. All this makes for an energy change of $\frac{1}{2} \cdot 4 \times 10^{14} \cdot 8 \times 10^{22} \cdot 4 \times 10^{-2}/16 \times 10^{16}$ J, or 4×10^{18} J, every 3×10^7 s, or about 0.1 TW. So if we implemented our shoreline trap-and-release scheme, it would effectively increase the rate at which tidal energy is removed from the Earth-Moon system by a factor of ten, thus speeding up the Moon's egress.

Wave

Waves come about every 10 s, raising a "breaker" about half a meter (peak of wave may be over 1 m, but most of mass is lower), and perhaps 2 m thick. So for each meter along the wave, a volume approximately 2 m^3 is raised 0.5 m for a potential energy of 10^4 J. Additionally, this mass of water is moving (once in breaker form) at maybe 3 m/s, for a kinetic energy of another 10^4 J. Waves every 10 s means each meter of projected coastline gets 2000 W of wave power. Seems like a lot: a hair dryer every meter! Let's say there is enough receptive coastline to circle the Earth three times (oblique or sheltered coastlines count less or not at all). This is 120,000 km for a total wave power of about 0.24 TW, or 2% of our worldly demand. Though 2000 W/m sounded like a big number, the linear rather than areal denominator made a huge difference.

Biofuels

We covered the scale of biological activity in Lecture 2, seeing that the total scale for oceans and land is about 70 TW (humans are 0.7 TW). Most of this feeds the chain of life on the planet (things eat each other). If we wanted to extract 15 TW from biofuels, we would effectively be commandeering one fifth of the life on the planet, which would then be unavailable for other, dependent life. So roughly speaking, we could only do so at the cost of effectively killing a significant fraction of the life on the planet.

Hydroelectric

If we consider North America, we have some big rivers emptying to the ocean that likely dominate the total hydro flow on our continent: Mississippi, Columbia, Yukon, St. Lawrence. How much flow? At their mouths we might estimate these rivers to be 1 km across, average of 10 m deep, and flowing at a depth-averaged speed of 2 m/s. Thus the flow is about 20,000 m^3/s for a big river. [Incidentally, the kinetic energy passing the mouth each second is $\sim 10^8$ W—though the Columbia River's Grand Coulee Dam produces 7 GW. Are our numbers off by two orders of magnitude somehow? I claim not—why?] Counting the big rivers, and assuming all the rest add up to about the same as the big ones, we get about 2×10^5 m^3/s flowing out of North America. If the average drop of rain falls on land with an elevation of 500 m above sea level, then the flow represents a gravitational potential energy per time of 10^{12} W. This counts every tributary and spring, as well as ground water flow, and some of the power naturally will go into turbulence/friction. (Otherwise wild rivers would emerge at the bottom of 1000 m mountains at a ripping speed of 140 m/s, or 300 m.p.h.!) So we might optimistically guess that only 10% of this is available in damable projects, or 0.1 TW for North America. In fact, this is not far from what we get: the U.S. uses 3 TW in total, and gets about 3%, or ~ 0.1 TW from hydro. We have developed all the low-hanging hydroelectric fruit already. We *might* be able to expand by a factor of two (capture 20% rather than 10%), but it will still be small beans.

Recommended Books & References

- Sustainable Energy: without the hot air, by David MacKay; free PDF online at `www.withouthotair.com`. Brilliant job assessing potential sources of renewables for the U.K.
- Energy: A Guidebook, by Janet Ramage; looks into alternatives not often discussed elsewhere (and their limitations).
- Carlos de Castro et al., Energy Policy, 39, 6677 (2011), "Global wind power potential: Physical and technological limits" `http://www.sciencedirect.com/science/article/pii/S0301421511004836`.